Exact formula of 3 flavor ν oscillation probability and its application to high energy astrophysical ν

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Contents

1. Introduction

1.1 Status of ν oscillation study

$N_\nu = 3$: $\nu_{atm} + \nu_{solar} + \nu_{reactor}$

Mixing matrix

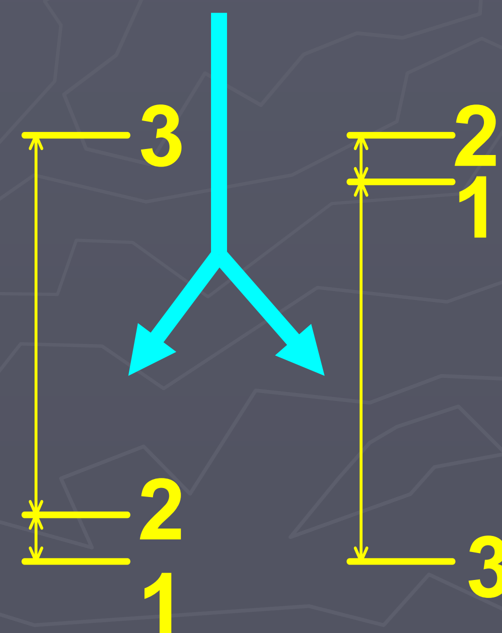
$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cong \begin{pmatrix} c_{12} & s_{12} & \varepsilon \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

- Both hierarchies are allowed

Mixing angles & mass squared differences

$$\theta_{12} \cong \pi/6, \quad \theta_{23} \cong \pi/4$$
$$|\theta_{13}| \cong |\varepsilon| \leq \sqrt{0.1}/2$$
$$\Delta m^2_{21} = 8\times 10^{-5}\,\mathrm{eV}^2$$
$$|\Delta m^2_{32}| = 2.5\times 10^{-3}\,\mathrm{eV}^2$$

- θ_{13} :only upper bound is known
- δ :undetermined



1.2 Exact oscillation probability in matter with constant density

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

$$\boxed{U\mathcal{E}U^{-1}} + \boxed{\mathcal{A}} = \tilde{U}\tilde{\mathcal{E}}\tilde{U}^{-1}$$

- $U\mathcal{E}U^{-1}$: oscillation in vacuum
- $\mathcal{A}$: matter effect

$$E_j \equiv \sqrt{\vec{p}^{\,2} + m_j^2}$$

$$\begin{aligned}
\mathcal{E} &\equiv \mathrm{diag}\,(E_1, E_2, E_3) \\
\mathcal{A} &\equiv \mathrm{diag}\,(A, 0, 0) \\
\tilde{\mathcal{E}} &\equiv \mathrm{diag}\left(\tilde{E}_1, \tilde{E}_2, \tilde{E}_3\right) \\
A &\equiv \sqrt{2}G_F N_e \\
\Delta\tilde{E}_{jk} &\equiv \tilde{E}_j - \tilde{E}_k
\end{aligned}$$

$$\begin{aligned}
P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} \quad &- \quad 4\sum_{j<k} \mathrm{Re}\left(\tilde{X}_j^{\alpha\beta}\tilde{X}_k^{\alpha\beta *}\right)\sin^2\left(\frac{\Delta\tilde{E}_{jk}L}{2}\right) \\
&+ \quad 2\sum_{j<k} \mathrm{Im}\left(\tilde{X}_j^{\alpha\beta}\tilde{X}_k^{\alpha\beta *}\right)\sin\left(\Delta\tilde{E}_{jk}L\right)
\end{aligned}$$

$$\tilde{X}_j^{\alpha\beta} \equiv \tilde{U}_{\alpha j}\tilde{U}^*_{\beta j}$$

Evaluation of $\tilde{U}_{\alpha j}$ is the hardest part

Kimura, Takamura, Yokomakura '02

Bilinear quantity $\tilde{X}_j^{\alpha\beta} \equiv \tilde{U}_{\alpha j}\tilde{U}^*_{\beta j}$ in matter

can be expressed as linear in bilinear quantity

$X_j^{\alpha\beta} \equiv U_{\alpha j}U^*_{\beta j}$ in vacuum:

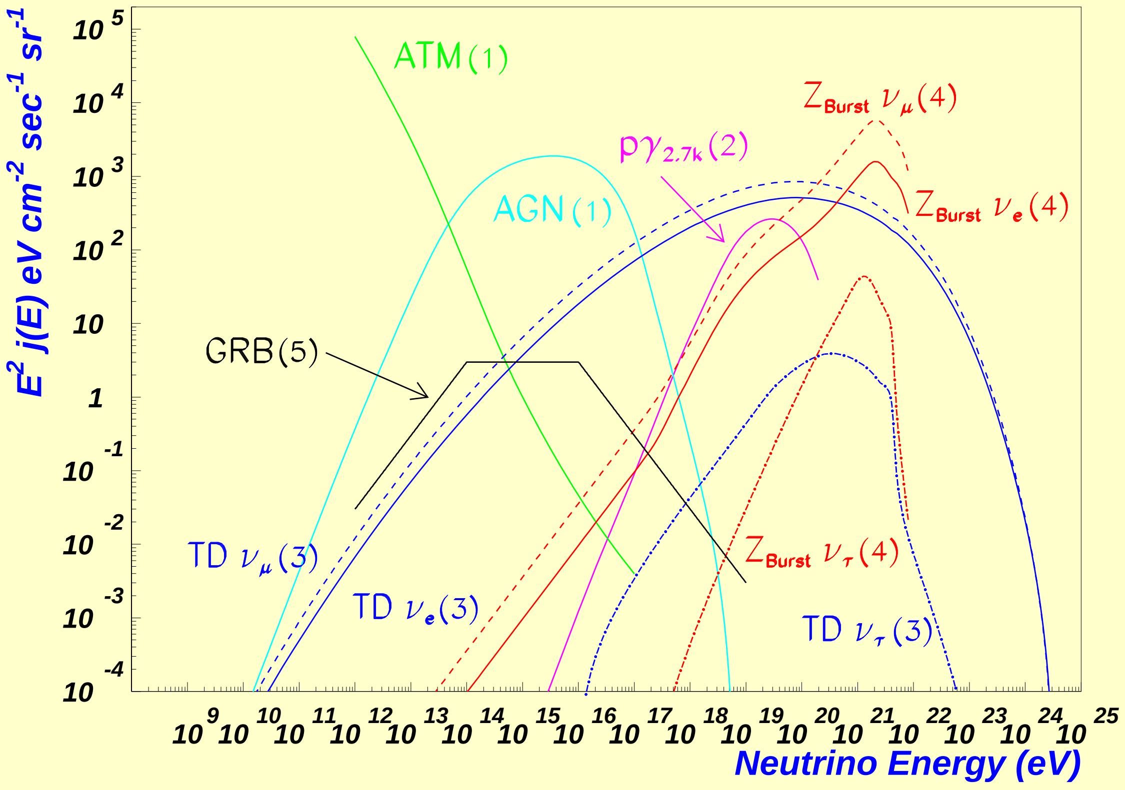
$$\tilde{X}_j^{\alpha\beta} = \sum_k F_{jk}^{\alpha\beta}(E_\ell, \tilde{E}_\ell, A) X_k^{\alpha\beta} + G_j^{\alpha\beta}(E_\ell, \tilde{E}_\ell, A)$$

simple known functions

- **Their derivation is complicated & confined only for N_ν=3 in constant matter → Another proof & generalization is given here**
- **There has been no example which was derived for the first time using KTY → a new result using KTY is presented**

1.3. High energy astrophysical ν

Flux of high energy cosmic ν from Active Galactic Nuclei or Gamma Ray Burst etc.



S/N ratio is expected to be large due to little background of atmospheric ν

- **Precise normalization of flux is not known**

→ **The ratio of different flavors is important quantity to observe**

- **Initial flux:**

Just like in ν_{atm}, the source of ν is π decay

➡ $F^0(\nu_e):F^0(\nu_\mu):F^0(\nu_\tau) \cong 1:2:0$

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\quad \hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\quad \hookrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

- **Observed flux on Earth:**

Due to ν oscillations

$|\theta_{13}| \ll 1,\ |\pi/4-\theta_{23}| \ll 1$ ➡ $F(\nu_e):F(\nu_\mu):F(\nu_\tau) \cong 1:1:1$

A few scenarios to predict deviation from 1:1:1 have been proposed

- **Standard flux + ν decay** — Beacom-Bell-Hooper-Pakvasa-Weiler '03

  α:1:1 (α=1.4~6)

- **Standard flux + pseudo-Dirac ν** — Beacom -Bell-Hooper-Learned-Pakvasa-Weiler'04

  α:1:1 (α=2/3~14/9)

- **Electromagnetic energy losses of π & μ** — Kashti-Waxman '05

  α:1:1 (α=1/1.8~1)

Here I will consider the possibility of standard flux + ν magnetic transitions + magnetic field

2. Another proof of Kimura-Takamura-Yokomakura formula

$$U\mathcal{E}U^{-1}+\mathcal{A} = \tilde{U}\tilde{\mathcal{E}}\tilde{U}^{-1}$$

$$\begin{aligned}
\mathcal{E} &\equiv \mathrm{diag}\left(E_1, E_2, E_3\right)\\
\mathcal{A} &\equiv \mathrm{diag}\left(A, 0, 0\right)\\
\tilde{\mathcal{E}} &\equiv \mathrm{diag}\left(\tilde{E}_1, \tilde{E}_2, \tilde{E}_3\right)\\
A &\equiv \sqrt{2}G_F N_e\\
\Delta\tilde{E}_{jk} &\equiv \tilde{E}_j - \tilde{E}_k
\end{aligned}$$

$$E_j \equiv \sqrt{\vec{p}^2 + m_j^2}$$

U **: Mixing matrix in vacuum**

$\tilde{U}$ **: Mixing matrix in matter**

Basic strategy is to take α,β component of both sides and to use the following identity:

$$\left(\tilde{U}\tilde{\mathcal{E}}^n\tilde{U}^{-1}\right)_{\alpha\beta} = \sum_j \left(\tilde{U}\right)_{\alpha j}\left(\tilde{E}_j\right)^n\left(\tilde{U}^\dagger\right)_{j\beta} = \sum_j \tilde{U}_{\alpha j}\left(\tilde{E}_j\right)^n\tilde{U}^*_{\beta j} = \sum_j \left(\tilde{E}_j\right)^n \tilde{X}_{\alpha\beta}$$

$$\left(\tilde{U}\tilde{U}^{-1}\right)_{\alpha\beta} = (1)_{\alpha\beta} \implies \sum_j \tilde{X}_j^{\alpha\beta} = \delta_{\alpha\beta}$$

$$\left(\tilde{U}\tilde{\mathcal{E}}\tilde{U}^{-1}\right)_{\alpha\beta} = \left(U\mathcal{E}U^{-1} + \mathcal{A}\right)_{\alpha\beta} \implies \sum_j \tilde{E}_j\tilde{X}_j^{\alpha\beta} = \sum_j E_j X_j^{\alpha\beta} + A\delta_{\alpha e}\delta_{e\beta}$$

$$\left(\tilde{U}\tilde{\mathcal{E}}^2\tilde{U}^{-1}\right)_{\alpha\beta} = \left[\left(U\mathcal{E}U^{-1} + \mathcal{A}\right)^2\right]_{\alpha\beta} \implies \sum_j \tilde{E}_j^2\tilde{X}_j^{\alpha\beta} = \sum_j E_j^2 X_j^{\alpha\beta} + \cdots$$

We get a linear equation for $\tilde{X}_j^{\alpha\beta}$

$$\begin{pmatrix} 1 & 1 & 1 \\ \tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 \\ \tilde{E}_1^2 & \tilde{E}_2^2 & \tilde{E}_3^2 \end{pmatrix} \begin{pmatrix} \tilde{X}_1^{\alpha\beta} \\ \tilde{X}_2^{\alpha\beta} \\ \tilde{X}_3^{\alpha\beta} \end{pmatrix} = \begin{pmatrix} \delta_{\alpha\beta} \\ \sum_j E_j X_j^{\alpha\beta} + A\delta_{\alpha e}\delta_{e\beta} \\ \sum_j E_j^2 X_j^{\alpha\beta} + \cdots \end{pmatrix}$$

We can solve the equation and reproduce KTY's result:

$$\tilde{X}_j^{\alpha\beta} = \sum_k F_{jk}^{\alpha\beta}(E_\ell, \tilde{E}_\ell, A)\, X_k^{\alpha\beta} + G_j^{\alpha\beta}(E_\ell, \tilde{E}_\ell, A)$$

It can be generalized to the case with adiabatically varying mass matrix in L=∞ limit:

$$
\begin{aligned}
i\frac{d}{dt}\psi(t) &= \tilde{U}(t)\,\tilde{\mathcal{E}}(t)\,\tilde{U}^{-1}(t) \\
\psi(t_2) &= \tilde{U}(t_2)\,\exp\left(-i\int_{t_1}^{t_2}\tilde{\mathcal{E}}(t)dt\right)\tilde{U}^{-1}(t_1)\psi(t_1) \\
A(\nu_\alpha\to\nu_\beta) &= \left[\tilde{U}(t_2)\,\exp\left(-i\int_{t_1}^{t_2}\tilde{\mathcal{E}}(t)dt\right)\tilde{U}^{-1}(t_1)\right]_{\beta\alpha} \\
&= \sum_j \tilde{U}(t_2)_{\beta j}\exp\left(-i\int_{t_1}^{t_2}\tilde{E}_j(t)dt\right)\tilde{U}(t_1)^*_{\alpha j} \\
P(\nu_\alpha\to\nu_\beta) &= |A(\nu_\alpha\to\nu_\beta)|^2 \\
&= \sum_{j,k}\tilde{U}(t_2)_{\beta j}\tilde{U}(t_2)^*_{\beta k}\tilde{U}(t_1)^*_{\alpha j}\tilde{U}(t_1)_{\alpha k}\exp\left(-i\int_{t_1}^{t_2}\Delta\tilde{E}_{jk}(t)dt\right) \\
&\to \sum_j\left|\tilde{U}(t_1)_{\alpha j}\right|^2\left|\tilde{U}(t_2)_{\beta j}\right|^2 \qquad \left(\exp\left(-i\int_{t_1}^{t_2}\Delta\tilde{E}_{jk}(t)dt\right)\to\delta_{jk}\right)
\end{aligned}
$$

Only in the limit $\int_{t_1}^{t_2}\Delta\tilde{E}_{jk}(t)dt\to\infty$ **, KTY method works**

Generalization of Kimura-Takamura-Yokomakura's result for the case with magnetic transitions & magnetic field

$$\mathcal{A} \equiv \mathrm{diag}\left(A_e + A_n, A_n, A_n\right)$$
$$A_e \equiv \sqrt{2} G_F N_e$$
$$A_n \equiv \frac{1}{\sqrt{2}} G_F N_n$$

$$\mathcal{L}_{mag} = \mu_{\alpha\beta} \overline{(\nu_{\alpha L})^c} \sigma_{\rho\sigma} F^{\rho\sigma} \nu_{\beta L} + h.c.$$

$\mu_{\alpha\beta}$: magnetic transitions

$$\mathcal{M} \equiv \begin{pmatrix} |\vec{p}| + \frac{1}{2|\vec{p}|} m^\dagger m + \mathcal{A} & |B_\perp| \mu \\ |B_\perp| \mu & |\vec{p}| + \frac{1}{2|\vec{p}|} m m^\dagger - \mathcal{A} \end{pmatrix} = \tilde{\mathcal{U}}\, \tilde{\mathcal{E}}_6\, \tilde{\mathcal{U}}^{-1}$$

$$\left(\tilde{U}\tilde{U}^{-1}\right)_{\alpha\beta} = (\mathbf{1})_{\alpha\beta}$$
$$\left(\tilde{\mathcal{U}}\, \tilde{\mathcal{E}}_6 \tilde{\mathcal{U}}^{-1}\right)_{\alpha\beta} = (\mathcal{M})_{\alpha\beta}$$
$$\left(\tilde{\mathcal{U}}\, (\tilde{\mathcal{E}}_6)^2 \tilde{\mathcal{U}}^{-1}\right)_{\alpha\beta} = \left(\mathcal{M}^2\right)_{\alpha\beta}$$
$$\left(\tilde{\mathcal{U}}\, (\tilde{\mathcal{E}}_6)^3 \tilde{\mathcal{U}}^{-1}\right)_{\alpha\beta} = \left(\mathcal{M}^3\right)_{\alpha\beta}$$
$$\left(\tilde{\mathcal{U}}\, (\tilde{\mathcal{E}}_6)^4 \tilde{\mathcal{U}}^{-1}\right)_{\alpha\beta} = \left(\mathcal{M}^4\right)_{\alpha\beta}$$
$$\left(\tilde{\mathcal{U}}\, (\tilde{\mathcal{E}}_6)^5 \tilde{\mathcal{U}}^{-1}\right)_{\alpha\beta} = \left(\mathcal{M}^5\right)_{\alpha\beta}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ \tilde{E}_1 & \tilde{E}_2 & \tilde{E}_3 & \tilde{E}_4 & \tilde{E}_5 & \tilde{E}_6 \\ \tilde{E}_1^2 & \tilde{E}_2^2 & \tilde{E}_3^2 & \tilde{E}_4^2 & \tilde{E}_5^2 & \tilde{E}_6^2 \\ \tilde{E}_1^3 & \tilde{E}_2^3 & \tilde{E}_3^3 & \tilde{E}_4^3 & \tilde{E}_5^3 & \tilde{E}_6^3 \\ \tilde{E}_1^4 & \tilde{E}_2^4 & \tilde{E}_3^4 & \tilde{E}_4^4 & \tilde{E}_5^4 & \tilde{E}_6^4 \\ \tilde{E}_1^5 & \tilde{E}_2^5 & \tilde{E}_3^5 & \tilde{E}_4^5 & \tilde{E}_5^5 & \tilde{E}_6^5 \end{pmatrix} \begin{pmatrix} \tilde{X}_1^{\alpha\beta} \\ \tilde{X}_2^{\alpha\beta} \\ \tilde{X}_3^{\alpha\beta} \\ \tilde{X}_4^{\alpha\beta} \\ \tilde{X}_5^{\alpha\beta} \\ \tilde{X}_6^{\alpha\beta} \end{pmatrix} = \begin{pmatrix} (\mathbf{1})_{\alpha\beta} \\ (\mathcal{M})_{\alpha\beta} \\ (\mathcal{M}^2)_{\alpha\beta} \\ (\mathcal{M}^3)_{\alpha\beta} \\ (\mathcal{M}^4)_{\alpha\beta} \\ (\mathcal{M}^5)_{\alpha\beta} \end{pmatrix}$$

In principle $\tilde{X}_j^{\alpha\beta}$ can be obtained from this

3. Flavor ratio of high energy astrophysical ν

In standard N_ν=3, when L→∞
oscillation probability in vacuum

Learned-Pakvasa '95

$$P_{\alpha\beta} = \sum_j |U_{\alpha j}|^2 |U_{\beta j}|^2$$

$$|U_{\alpha j}|^2 \cong \begin{pmatrix} c_{12}^2 & s_{12}^2 & 0 \\ s_{12}^2/2 & c_{12}^2/2 & 1/2 \\ s_{12}^2/2 & c_{12}^2/2 & 1/2 \end{pmatrix}$$

$$F(\nu_e) = F^0(\nu_e)(P_{ee} + 2P_{\mu e}) = F^0(\nu_e)(1\text{-}P_{\tau e} + P_{\mu e}) = 1$$

$$F(\nu_\mu) = F^0(\nu_e)(P_{e\mu} + 2P_{\mu\mu}) = F^0(\nu_e)(1\text{-}P_{\tau\mu} + P_{\mu\mu}) = 1$$

$$F(\nu_\tau) = F^0(\nu_e)(P_{e\tau} + 2P_{\mu\tau}) = F^0(\nu_e)(1\text{-}P_{\tau\tau} + P_{\mu\tau}) = 1$$

$$F(\nu_\alpha) = F^0(\nu_e)P_{e\alpha} + F^0(\nu_\mu)P_{\mu\alpha} = F^0(\nu_e)(P_{e\alpha} + 2P_{\mu\alpha})$$

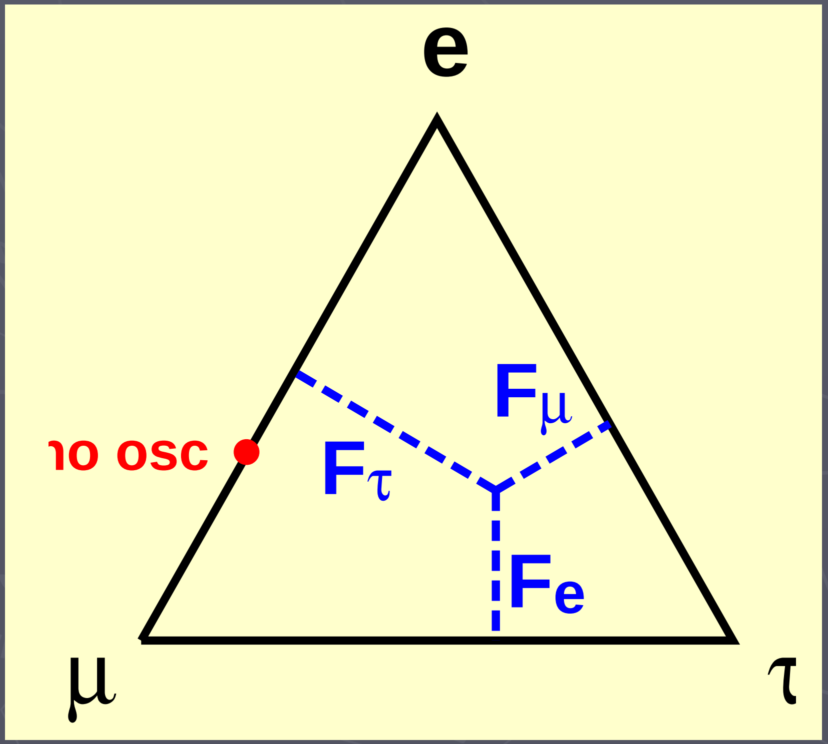
$$P_{e\alpha} + 2P_{\mu\alpha} = (P_{e\alpha} + P_{\mu\alpha}) + P_{\mu\alpha} = 1\text{-}P_{\tau\alpha} + P_{\mu\alpha} = 1$$

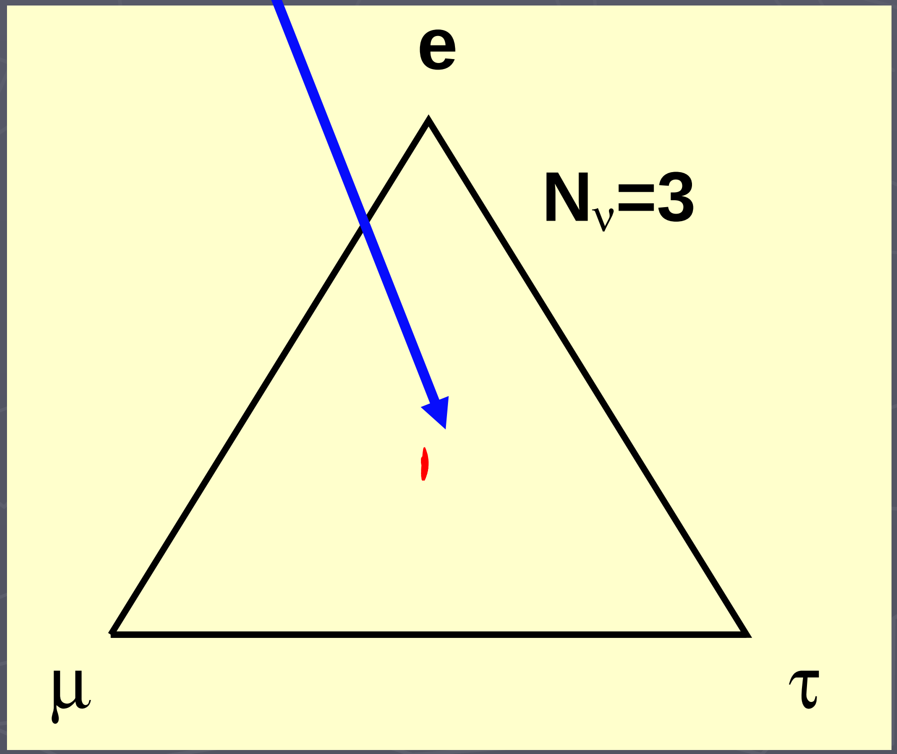
CHOOZ+ν_{atm}: $|\theta_{13}|$<<1

ν_{atm}: $|\pi/4 - \theta_{23}|$<<1

Deviation from 1:1:1 is small

Athar-Jezabek-OY '00





For simplicity we consider the Majorana case with CP invariance

- Majorana ν: → $\mu_{\alpha\alpha}$ =0 , $\mu_{\alpha\beta}$ = -$\mu_{\beta\alpha}$ =pure imaginary
- no matter effect: A=0
- KM–like CP phase δ decouples (←$|\theta_{13}|$<<1)
- no CP phase from the charged lepton sector: β', γ' =0

$$
\begin{aligned}
m &= V^* \,\mathrm{diag}(m_j)\, V^\dagger \\
V &= e^{i\alpha} \boxed{e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}}\, U \boxed{e^{-i\gamma\lambda_8} e^{-i\beta\lambda_3}} \\
m^\dagger m &= V \,\mathrm{diag}(m_j^2)\, V^\dagger = e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}\, U \,\mathrm{diag}(m_j^2)\, U^\dagger \, e^{-i\gamma'\lambda_8} e^{-i\beta'\lambda_3} \\
m m^\dagger &= V^* \,\mathrm{diag}(m_j^2)\, V^T = e^{-i\gamma'\lambda_8} e^{-i\beta'\lambda_3}\, U^* \,\mathrm{diag}(m_j^2)\, U^T \, e^{i\beta'\lambda_3} e^{i\gamma'\lambda_8}
\end{aligned}
$$

NB: β, γ (Majorana CP phases) don't appear in the eq.

In this case analysis of 6x6 matrix is reduced to that of 3x3:

$$\begin{pmatrix} U\mathcal{E}U^{-1} & |B_\perp|\mu \\ |B_\perp|\mu & U\mathcal{E}U^{-1} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} U\mathcal{E}U^{-1} + |B_\perp|\mu & 0 \\ 0 & U\mathcal{E}U^{-1} - |B_\perp|\mu \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

We consider the situation in which

B(t=0)❍0 → B(t=L)=0

occurs adiabatically

Assuming adiabatic approximation (|B(t=0)|>0 →B(t=L)=0), oscillation probability in the limit L→∞ can be analytically expressed :

$$U\mathcal{E}U^{-1} + |B_\perp|\mu = U\mathcal{E}U^{-1} + i|B_\perp|Im(\mu) = \tilde{U}\,\tilde{\mathcal{E}}\,\tilde{U}^{-1}$$

$$P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 2\sum_j \left[Re\tilde{U}_{\alpha j}(t=0)\right]^2 \left|\tilde{U}_{\beta j}(t=L)\right|^2$$

$$P(\nu_\alpha \to \bar{\nu}_\beta) = P(\bar{\nu}_\alpha \to \nu_\beta) = 2\sum_j \left[Im\tilde{U}_{\alpha j}(t=0)\right]^2 \left|\tilde{U}_{\beta j}(t=L)\right|^2$$

$$P(\nu_\alpha \to \nu_\beta) + P(\bar{\nu}_\alpha \to \nu_\beta) = P(\nu_\alpha \to \bar{\nu}_\beta) + P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 2\sum_j \tilde{X}_j^{\alpha\alpha}(t=0) X_j^{\beta\beta}$$

$$F(\nu_\alpha) = F^0(\nu_e)\sum_j X_j^{\alpha\alpha}(\tilde{X}_j^{\mu\mu} - \tilde{X}_j^{\tau\tau})$$

$$X_j^{\mu\mu} = X_j^{\tau\tau} \rightarrow F(\nu_\mu) = F(\nu_\tau)$$

$$F(\nu_e) : F(\nu_\mu) : F(\nu_\tau) = \alpha : 1 : 1$$

The ratio of $\nu_e + \overline{\nu}_e$ can be analytically expressed :

$$
\begin{aligned}
\frac{F(\nu_e) + F(\bar{\nu}_e)}{2F^0(\nu_e)} &= 1 + \sum_j |U_{ej}|^2 \left[|\tilde{U}_{\mu j}(t=0)|^2 - |\tilde{U}_{\tau j}(t=0)|^2 \right] \\
&= 1 + \frac{|B_\perp|^2 (\mu_{e\mu}^2 - \mu_{e\tau}^2)}{\Delta \tilde{E}_{21}} \left(\frac{c_{12}^2}{\Delta \tilde{E}_{31}} - \frac{s_{12}^2}{\Delta \tilde{E}_{32}} \right) \\
&= 1 + \frac{(w^2 - v^2)/4}{(\Delta E_{31} + \Delta E_{21})^2/3 - \Delta E_{31}\Delta E_{21} + u^2 + v^2 + w^2} \\
&\quad \times \frac{1}{\sin\phi} \left(\frac{c_{12}^2}{\sin(\phi - \pi/3)} + \frac{s_{12}^2}{\sin(\phi + \pi/3)} \right)
\end{aligned}
$$

$$
\begin{aligned}
\phi &\equiv \frac{1}{3} \cos^{-1} \frac{A}{B}, \quad u \equiv |B_\perp| \mu_{\mu\tau}, \quad v \equiv |B_\perp| \mu_{e\tau}, \quad w \equiv |B_\perp| \mu_{e\mu} \\
A &\equiv \left(\frac{\Delta E_{31} + \Delta E_{21}}{3} \right)^3 - \frac{1}{6} (\Delta E_{31} + \Delta E_{21})(\Delta E_{31}\Delta E_{21} - u^2 - v^2 - w^2) \\
&\quad - \frac{1}{2} \left\{ (v - w)^2 \Delta E_{31} + \left[\sqrt{2} s_{12} u - c_{12}(v + w) \right]^2 \Delta E_{21} \right\} \\
B &\equiv \left[\left(\frac{\Delta E_{31} + \Delta E_{21}}{3} \right)^2 - \frac{\Delta E_{31}\Delta E_{21}}{3} + \frac{1}{3}(u^2 + v^2 + w^2) \right]^{3/2}
\end{aligned}
$$

The effect of the magnetic transitions becomes largest when $|\mu_{\mu\tau}| \gg |\mu_{e\mu}|, |\mu_{e\tau}|, |\Delta E_{jk}|/|B_\perp|$

In this case the ratio $\frac{F(\nu_e) + F(\bar{\nu}_e)}{2F^0(\nu_e)}$ **$\rightarrow 3/2$**

Unfortunately, for the above condition to be satisfied, (ν energy) $\times$ (magnetic field at production point) has to be unrealistically large:

$$(E_\nu/1\text{TeV})(|B_\perp|/1\text{G}) > 1$$

assuming $|\mu_{\mu\tau}| \cong 10^{-10}\,\mu_B$ But if this condition is satisfied, then nontrivial energy dependence of the ratio $\frac{F(\nu_e) + F(\bar{\nu}_e)}{2F^0(\nu_e)}$ **should be observed**

4. Summary

- **Simple proof and generalization of Kimura-Takamura-Yokomakura formula is given**
  - **Generalization to the case with magnetic transitions**
  - **Generalization to the case with non-constant adiabatically varying matter effect and/or magnetic field**
- **Flavor ratio of high energy astrophysical ν is expressed analytically taking into account of N_ν=3 mixing matrix under certain assumptions (Majorana ν & CP invariance)**

- **Further situations to be considered in future:**
  - **Possibility of non-adiabatic transition**
  - **Effect of phases from the charged lepton sector: β', γ'**