

Systematic limits of multi reactors and detectors

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I. One detector + reactors

II. Multi detectors + reactors



**Work in collaboration with
H. Sugiyama, F. Suekane,
G. Horton-Smith**

III. Spectrum analysis



**Work in collaboration with
H. Sugiyama**

Assumptions throughout this talk:

- Systematic errors only (=in the ∞ statistical limit)

$(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}}$: limit on $\sin^2 2\theta_{13}$
w/o stat error

- Background not considered (no difference in calibration for # (reactors) > 1)

- **Identical** systematic errors (in **Kashiwazaki**)

from detectors $\left\{ \begin{array}{ll} \sigma_c & \text{correlated} \quad \sim 1.6\% \\ \sigma_u & \text{uncorrelated} \quad \sim 0.6\% \end{array} \right.$

from reactors $\left\{ \begin{array}{ll} \sigma_c^{(r)} & \text{correlated} \quad \sim 2.5\% \\ \sigma_u^{(r)} & \text{uncorrelated} \quad \sim 2.3\% \end{array} \right.$

I. One detector + multi reactors

(1) 1 detector + 1 reactor

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \left[\frac{M - T(1 + \alpha_c + \alpha_c^{(r)} + \alpha_u^{(r)})}{T \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \left(\frac{\alpha_u^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$
$$= \frac{(M/T - 1)^2}{\sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2} \equiv \frac{(M/T - 1)^2}{\sigma_{\text{eff}}^2}$$



ν_e



M: measured # (events), **T**: theoretical # (events)

Limit on $\sin^2 2\theta$

$$T = \int \varepsilon(E) \sigma(E) f(E) dE$$

$$M = \int \varepsilon(E) \sigma(E) f(E) P(E) dE$$

$$T - M = \sin^2 2\theta \int \varepsilon(E) \sigma(E) f(E) \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) dE$$

$$\frac{T - M}{T} = \sin^2 2\theta \left\langle \sin^2 \left(\frac{\Delta m^2 L}{4E} \right) \right\rangle \cong 0.8 \sin^2 2\theta$$

$$\chi^2 = \frac{(0.8 \sin^2 2\theta)^2}{\sigma_{\text{eff}}^2}, \quad \chi^2|_{90\%CL} = 2.7$$

$$\left(\sin^2 2\theta \right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.8} \sigma_{\text{eff}}$$

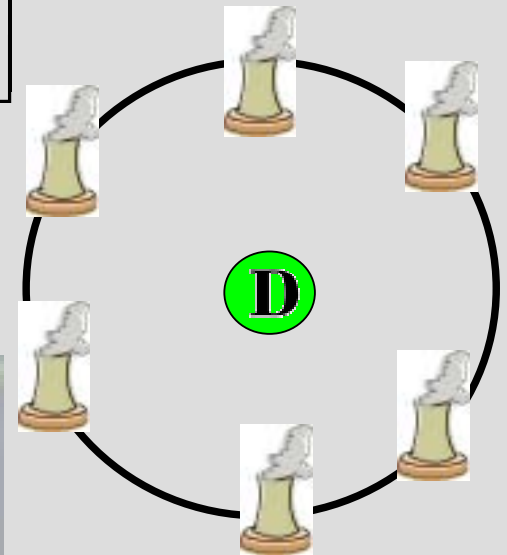
For $L=1.7\text{km}$,
 $\Delta = 2.5 \times 10^{-3} \text{eV}^2$

(2) 1 detector +N reactors

$$\begin{aligned} T &= \sum_{a=1}^N T_a \rightarrow \sum_{a=1}^N T_a (1 + \alpha_c + \alpha_c^{(r)} + \alpha_{ua}^{(r)}) \\ &= T \left[1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^N \left(\frac{T_a}{T} \right) \alpha_{ua}^{(r)} \right] \end{aligned}$$

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \frac{\left[M - T (1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^N \left(\frac{T_a}{T} \right) \alpha_{ua}^{(r)}) \right]^2}{T \sigma_u} + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^N \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$

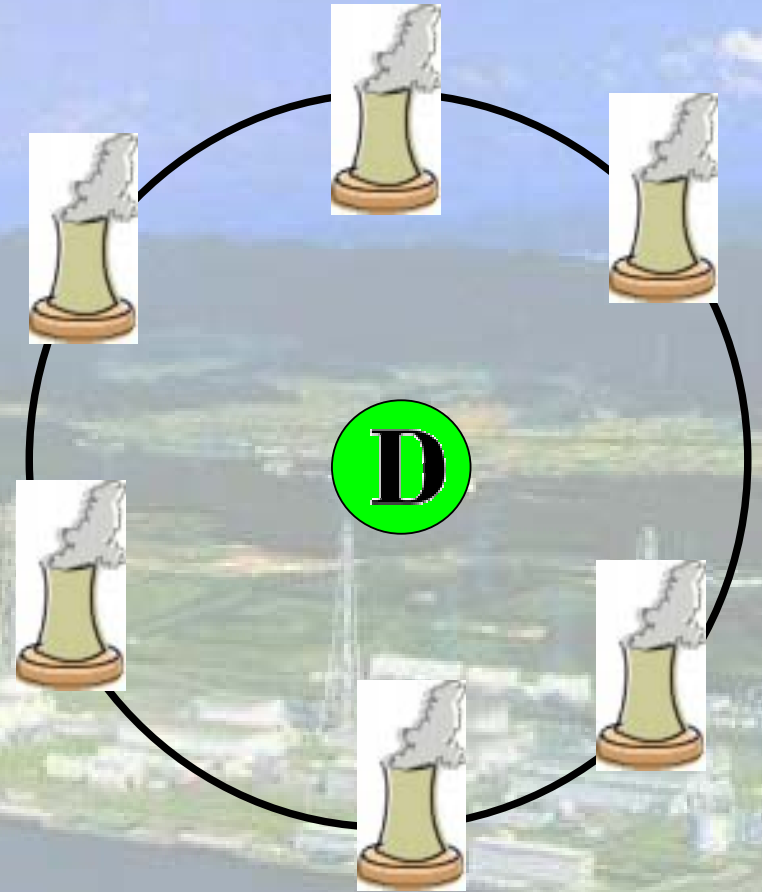
$$\sigma_{\text{eff}}^2 = \sigma_u^2 + \sigma_c^2 + \left(\sigma_c^{(r)} \right)^2 + \left(\sigma_u^{(r)} \right)^2 \sum_{a=1}^N \left(\frac{T_a}{T} \right)^2$$



Assume equal yield
from each reactor:
Then

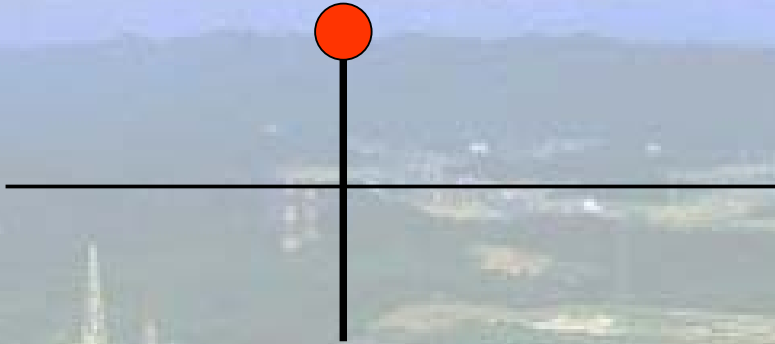
$$\frac{T_a}{T} = \frac{T_a}{\sum_{a=1}^N T_a} = \frac{1}{N}$$

$$\sum_{a=1}^N \left(\frac{T_a}{T} \right)^2 = \sum_{a=1}^N \frac{1}{N^2} = \frac{1}{N}$$

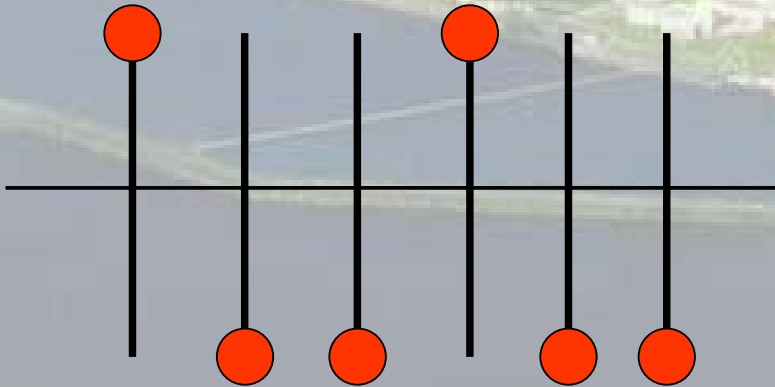


$$\sigma_{\text{eff}}^2 = \sigma_u^2 + \sigma_c^2 + \left(\sigma_c^{(r)} \right)^2 + \frac{1}{N} \left(\sigma_u^{(r)} \right)^2$$

Independent fluctuations of N variables



$N=1$: Fluctuation is large



$N > 1$: Fluctuation is small in average

II. Multi detectors + multi reactors

(1) 2 detectors + 1 reactor

$$\mathbf{x}^2 = \min_{\alpha\text{'s}} \left\{ \left[\frac{\mathbf{M}^n - \mathbf{T}^n (1 + \alpha_c + \alpha_c^{(r)} + \alpha_u^{(r)})}{\mathbf{T}^n \sigma_u} \right]^2 + \left[\frac{\mathbf{M}^f - \mathbf{T}^f (1 + \alpha_c + \alpha_c^{(r)} + \alpha_u^{(r)})}{\mathbf{T}^f \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \left(\frac{\alpha_u^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$

$$= (\mathbf{y}^n, \mathbf{y}^f) \mathbf{V}^{-1} \begin{pmatrix} \mathbf{y}^n \\ \mathbf{y}^f \end{pmatrix}, \quad \mathbf{y}^n \equiv \frac{\mathbf{M}^n - \mathbf{T}^n}{\mathbf{T}^n}, \quad \mathbf{y}^f \equiv \frac{\mathbf{M}^f - \mathbf{T}^f}{\mathbf{T}^f},$$

$$\mathbf{V} \equiv \begin{pmatrix} \sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 & \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 \\ \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 & \sigma_u^2 + \sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2 \end{pmatrix}$$



ν_e



After diagonalization of V

$$\chi^2 = \frac{(\mathbf{y}^f - \mathbf{y}^n)^2}{2\sigma_u^2} + \frac{(\mathbf{y}^f + \mathbf{y}^n)^2}{2\sigma_u^2 + 4[\sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2]}$$



No correlated error

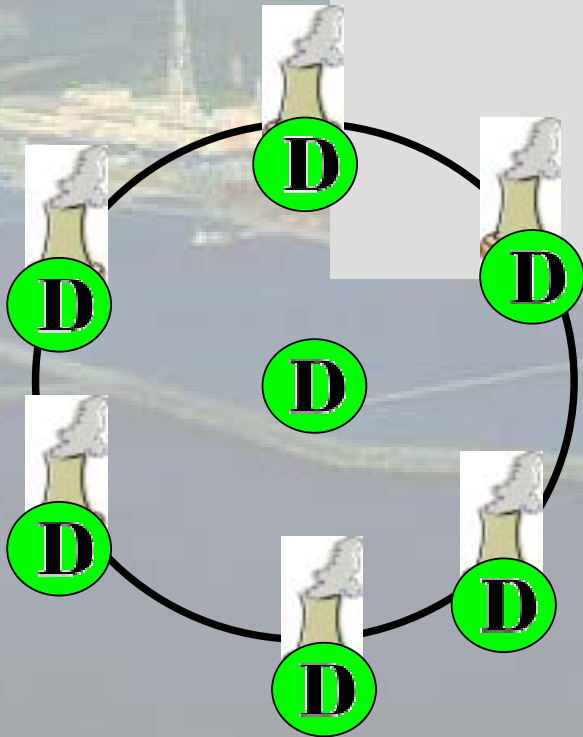
$$\sigma_{\text{eff}} \cong \sqrt{2}\sigma_u \left[1 + \frac{1}{1 + 2(\sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2) / \sigma_u^2} \right]^{-1/2}$$



σ_{eff} is dominated by
the **uncorrelated** error

(2) (N+1) detectors+N reactors

$$\chi^2 = \min_{\alpha\text{'s}} \left\{ \sum_{i=1}^{N+1} \left[\frac{\mathbf{M}^i - \mathbf{T}^i(\mathbf{1} + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^N \left(\frac{\mathbf{T}_a^i}{\mathbf{T}^i} \right) \alpha_{ua}^{(r)})}{\mathbf{T}^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^N \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$



$$\sigma_{\text{eff}} = \sqrt{1 + \frac{1}{N}} \sigma_u \left[1 + \frac{1/N}{\underbrace{1 + (N+1)(\sigma_c^2 + (\sigma_c^{(r)})^2 + (\sigma_u^{(r)})^2/N)}_{\text{small}}} / \sigma_u^2 \right]^{-1/2}$$

$(1 + 1/N)^{1/2}$: σ_{eff} improves as $N \rightarrow \infty$
 (more info w/ more detectors)

$1/N$: slightly better from cancellation of uncorrelated error due to multi reactors

(3) Ideal limit of Kashiwazaki-Kariwa (3 detectors+2 reactors)

$$P=3P_0$$



D

$$x^2 = \min_{\alpha\text{'s}} \left\{ \sum_{i=1}^3 \left[\frac{M^i - T^i(1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^2 \left(\frac{T_a^i}{T^i} \right) \alpha_{ua}^{(r)})}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^2 \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$



D

$$P=4P_0$$

D

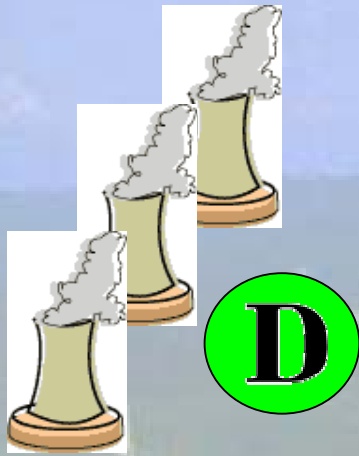
$$\sigma_{\text{eff}} \cong \frac{\sqrt{74}}{7} \sigma_u$$

$$L_f = 1.3 \text{ km}$$

$$L_n = 0.3 \text{ km}$$

$$(\sin^2 2\theta)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.64} \sigma_{\text{eff}} = 0.018$$

(4) Actual Kashiwazaki-Kariwa



$$x^2 = \min_{\alpha's} \left\{ \sum_{i=1}^3 \left[\frac{M^i - T^i(1 + \alpha_c + \alpha_c^{(r)} + \sum_{a=1}^7 \left(\frac{T_a^i}{T^i} \right) \alpha_{ua}^{(r)})}{T^i \sigma_u} \right]^2 + \left(\frac{\alpha_c}{\sigma_c} \right)^2 + \left(\frac{\alpha_c^{(r)}}{\sigma_c^{(r)}} \right)^2 + \sum_{a=1}^7 \left(\frac{\alpha_{ua}^{(r)}}{\sigma_u^{(r)}} \right)^2 \right\}$$



$$\frac{\sigma_{\text{eff}}|_{\text{actual KK}}}{\sigma_{\text{eff}}|_{\text{ideal KK}}} = 1 - 7 \times 10^{-3}$$

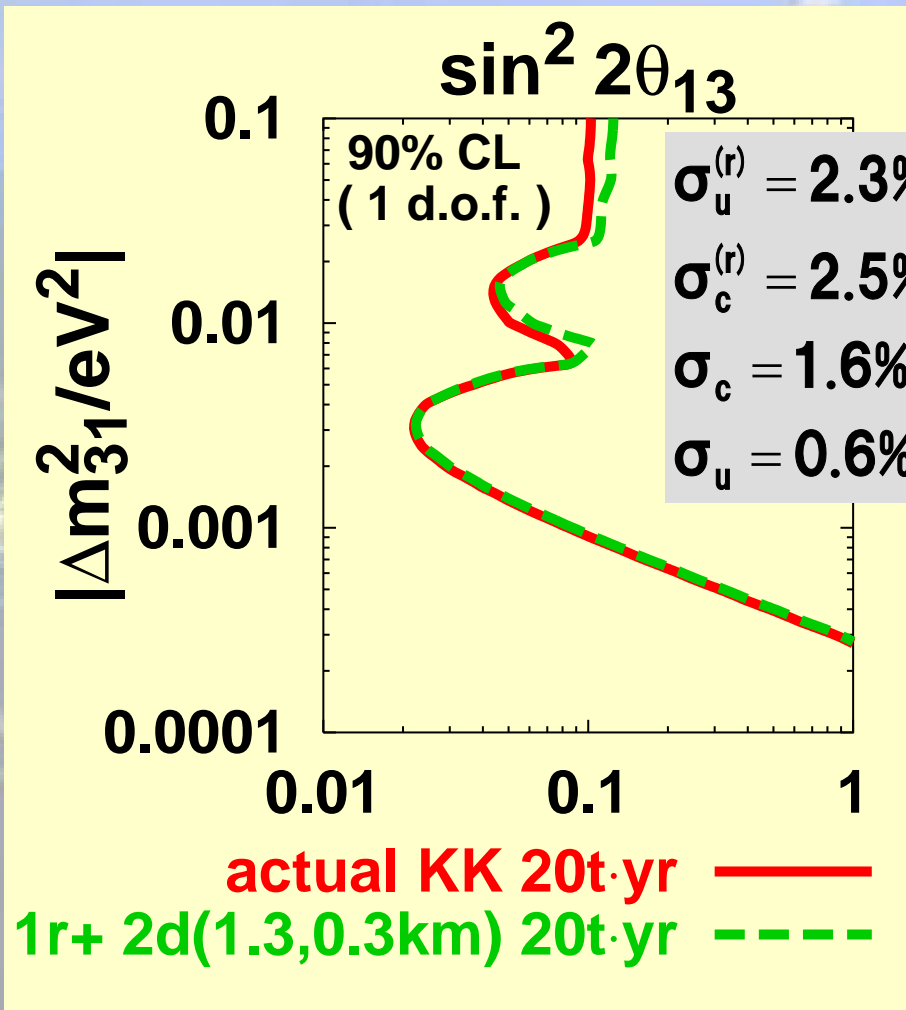
Therefore also for actual KK

$$\left(\sin^2 2\theta \right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.64} \sigma_{\text{eff}} = \mathbf{0.018}$$

σ_{eff} of actual KK plan is almost the same as that of the ideal case.

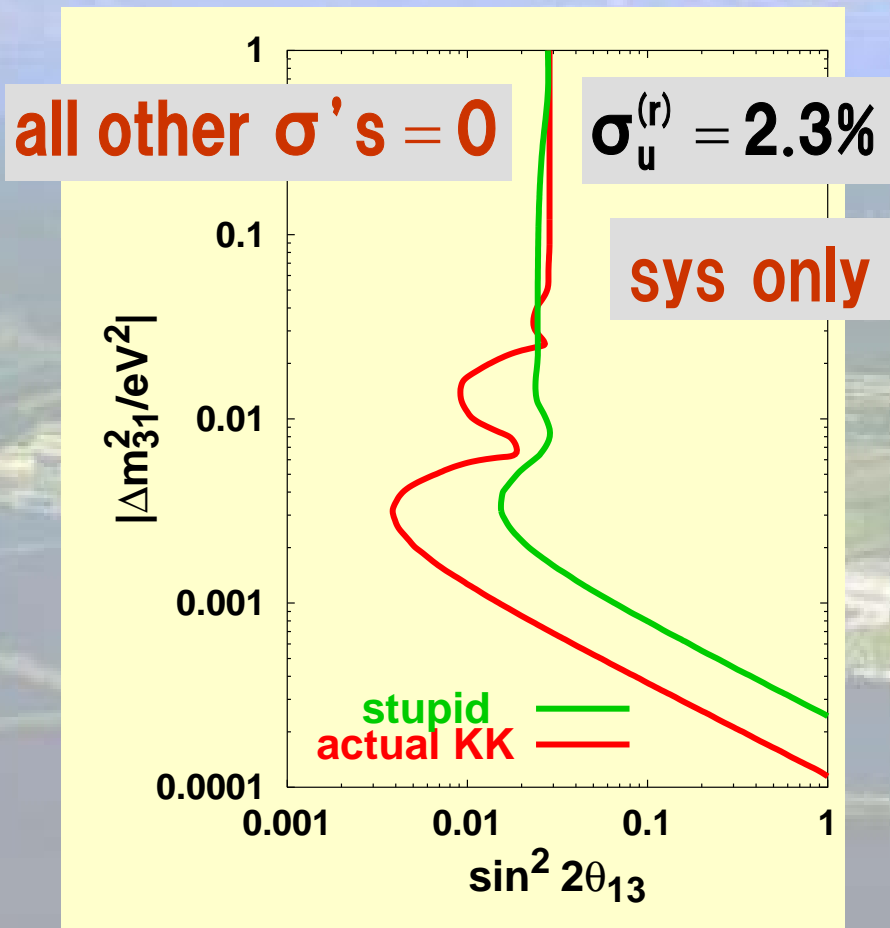
It is slightly better because of 5 more reactors.

Comparison of (1reactor+2detectors) and KK

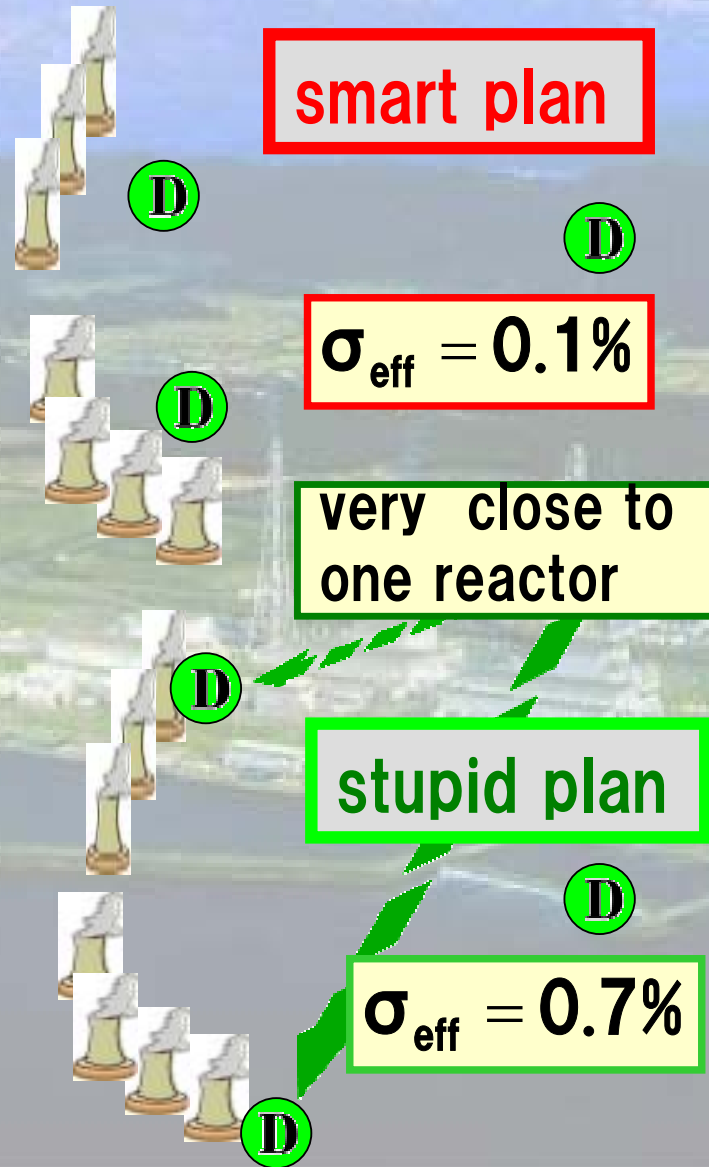


KK is slightly better because of one more detector.

Comparison of locations of near detectors



KK plan is almost optimal.

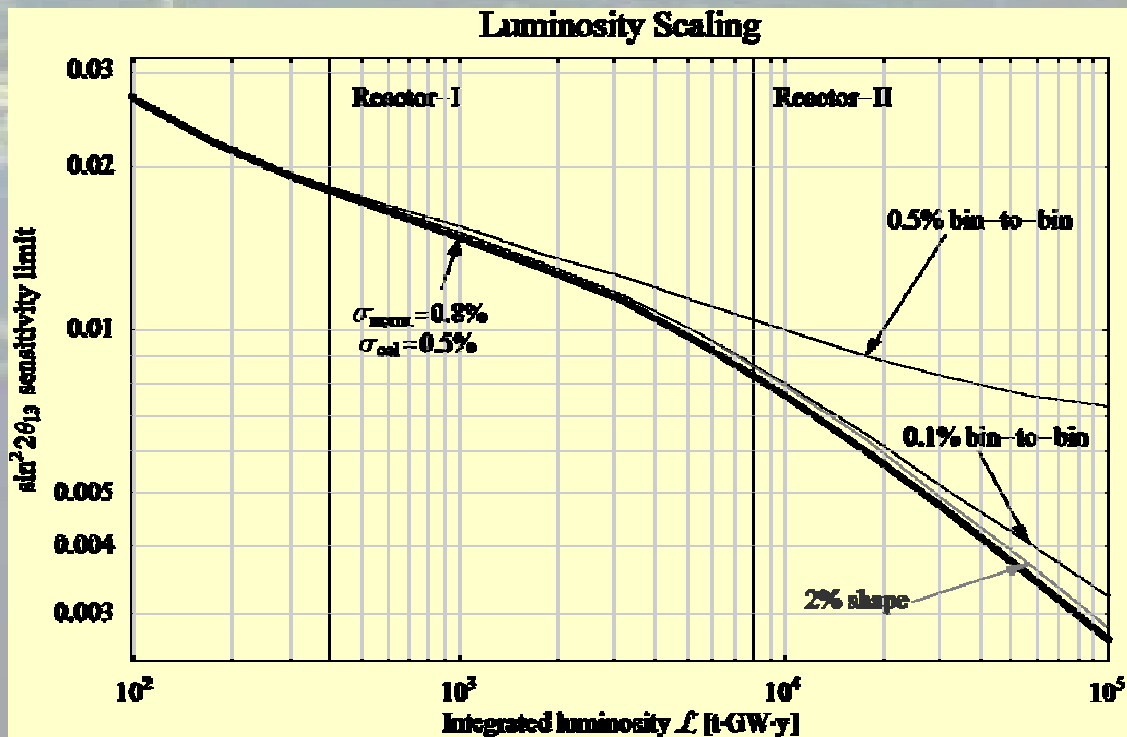


III. Energy spectrum (with H. Sugiyama)

Examination of the result in

Huber-Lindner-Schwetz - Winter, NP B665, 487 ('03)

By building Super KamLAND, is it really possible to reach $\sin^2 2\theta_{13} \sim 0.003$?



2 detectors + 1 reactor

$$\mathbf{x}^2 = \min_{\alpha\text{'s}} \left\{ \sum_j \left[\frac{\mathbf{M}_j^n - \mathbf{T}_j^n (1 + \alpha^{(r)} + \alpha_j + \alpha_n)}{\mathbf{T}_j^n \sigma_u} \right]^2 + \sum_j \left[\frac{\mathbf{M}_j^f - \mathbf{T}_j^f (1 + \alpha^{(r)} + \alpha_j + \alpha_f)}{\mathbf{T}_j^f \sigma_u} \right]^2 + \left(\frac{\alpha^{(r)}}{\sigma^{(r)}} \right)^2 + \sum_j \left(\frac{\alpha_j}{\sigma_{\text{shape}}} \right)^2 + \left(\frac{\alpha_n}{\sigma_c} \right)^2 + \left(\frac{\alpha_f}{\sigma_c} \right)^2 \right\}$$

$$\begin{array}{l} \sigma^{(r)} \leftarrow \sigma_a, \quad \sigma_c \leftarrow \sigma_b \\ \sigma_u \leftarrow \sigma_{\text{exp}}, \quad \sigma_{\text{shape}} \leftarrow \sigma_{\text{shape}} \end{array}$$

※Error of energy calibration gives little contribution and omitted for simplicity.

Assumptions: Uncorrelated bin-to-bin error

$n = \#$ (bins)

is independent of bin: $\sigma_{uj} = \sigma_u$

$$\sigma_{\text{eff}} \approx \sqrt{\frac{a_n}{n}} \sigma_u \left[1 + \frac{\sigma_u^2}{\sigma_u^2 + 2\sigma_{\text{shape}}^2} + b_n \left(\frac{\sigma_u^2}{\sigma_u^2 + n\sigma_c^2} + \frac{\sigma_u^2}{\sigma_u^2 + 2\sigma_{\text{shape}}^2 + n\sigma_c^2 + 2n(\sigma^{(r)})^2} \right) \right]^{\frac{1}{2}}$$

n	a_n	b_n
2	151	44
4	76	20
6	60	15
8	51	12
16	41	10
≥ 30	~ 40	~ 9

For simplicity, assume: $\sigma_{\text{shape}} \rightarrow 0$, $\sigma^{(r)} \gg \sigma_c, \sigma_u$

$$\sigma_{\text{eff}} \cong \sqrt{\frac{a_n}{2n}} \sigma_u \left[1 + \frac{b_n}{2} \frac{\sigma_u^2}{\sigma_u^2 + n \sigma_c^2} \right]^{-\frac{1}{2}}$$

Consider two cases:

(i) $\sigma_c = \sigma_u = 0.6\%$

$$\rightarrow \sigma_{\text{eff}} \cong \sqrt{\frac{a_n}{2n}} \sigma_u \left(1 + \frac{b_n/2}{n+1} \right)^{-\frac{1}{2}}$$

Contribution of σ_c is **not** negligible for $n < 10$.


(ii) $\sigma_u = 0.1\% \ll \sigma_c = 0.6\%$

$$\rightarrow \sigma_{\text{eff}} \cong \sqrt{\frac{a_n}{2n}} \sigma_u \xrightarrow{n=62} (\sin^2 2\theta)_{\text{limit}}^{\text{sys only}} = \sqrt{2.7} \sigma_{\text{eff}} \cong 0.0009 \text{ !?}$$

$n = \# \text{ (bin)}$	$(a_n / 2n)^{1/2}$	$b_n / (2n+2)$
2	6.2	7.3
4	3.1	2.0
6	2.2	1.1
8	1.8	0.7
16	1.1	0.3
62	0.6	0.1

σ_{eff} is determined by σ_u .

In any case, unless $\sigma_u \gg \sigma_c$ is satisfied, σ_u gives dominant contribution to σ_{eff} & $(\sin^2 2\theta)_{\text{limit}}^{\text{sys only}}$



Realistic σ_u has to be estimated carefully.

IV. Summary

(1) With 1 detector: limit on $\sin^2 2\theta$ is dominated by the correlated error, but the uncorrelated error from reactors decreases with multi reactors. ($N = \#$ (reactors))

$$\left(\sin^2 2\theta\right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.8} \sqrt{\sigma_u^2 + \sigma_c^2 + (\sigma_u^{(r)})^2 / N}$$

(2) With multiple detectors: limit on $\sin^2 2\theta$ is dominated by the uncorrelated error, and $\#$ (reactors) is irrelevant. ($N = \#$ (detectors))

$$\left(\sin^2 2\theta\right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.8} \sqrt{1 + \frac{1}{N}} \sigma_u \left(1 + \text{small correction}\right)$$

(2') Kashiwazaki plan does not have any disadvantage over plans w/ 1 or 2 reactor (s) .

$$\left(\sin^2 2\theta\right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.64} \sigma_{\text{eff}} = 0.018$$

$$\begin{aligned} \left(\sin^2 2\theta\right)_{\text{sens.}} &= \frac{\sqrt{2.7}}{0.64} \sqrt{\sigma_{\text{eff}}^2 + \sigma_{\text{stat}}^2} \\ &= 0.020 \end{aligned}$$

For 20ton·yr: $\sigma_{\text{stat}}^2 = 1/60000 = (0.4\%)^2$

(3) The energy spectrum analysis:

For larger $n \equiv \#$ (bins), limit on $\sin^2 2\theta$ is dominated by the uncorrelated bin-to-bin error σ_u , and one has to estimate realistic σ_u .

$$\left(\sin^2 2\theta\right)_{\text{limit}}^{\text{sys-only}} = \frac{\sqrt{2.7}}{0.8} \sqrt{\frac{a_n}{n}} \sigma_u [1 + \dots]$$

Appendix

$$\mathbf{V}_{\text{ideal}} = \sigma_u^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + [\sigma_c^2 + (\sigma_c^{(r)})^2] \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \\ + (\sigma_u^{(r)})^2 \begin{pmatrix} 1/4 & 0 & 1/7 \\ 0 & 1/3 & 1/7 \\ 1/7 & 1/7 & 1/7 \end{pmatrix}$$

$$\mathbf{V}_{\text{actual}} = \sigma_u^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + [\sigma_c^2 + (\sigma_c^{(r)})^2] \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \\ + (\sigma_u^{(r)})^2 \begin{pmatrix} 0.221 & 0.041 & 0.149 \\ 0.041 & 0.291 & 0.134 \\ 0.149 & 0.134 & 0.144 \end{pmatrix}$$