Toward exploring U_{e3}

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"To be renamed as 首都大学東京"

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Many thanks to H. Sugiyama for discussions

1. Introduction

Even if we know $P(\nu_\mu \to \nu_e)$ and $P(\overline{\nu_\mu} \to \overline{\nu_e})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of θ_{13}, sign(Δm^2_{31}) and δ is difficult because of the 8-fold parameter degeneracy.

- intrinsic (δ, θ_{13}) degeneracy
- $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy
- $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

Formalism in this talk

Notations in this talk:

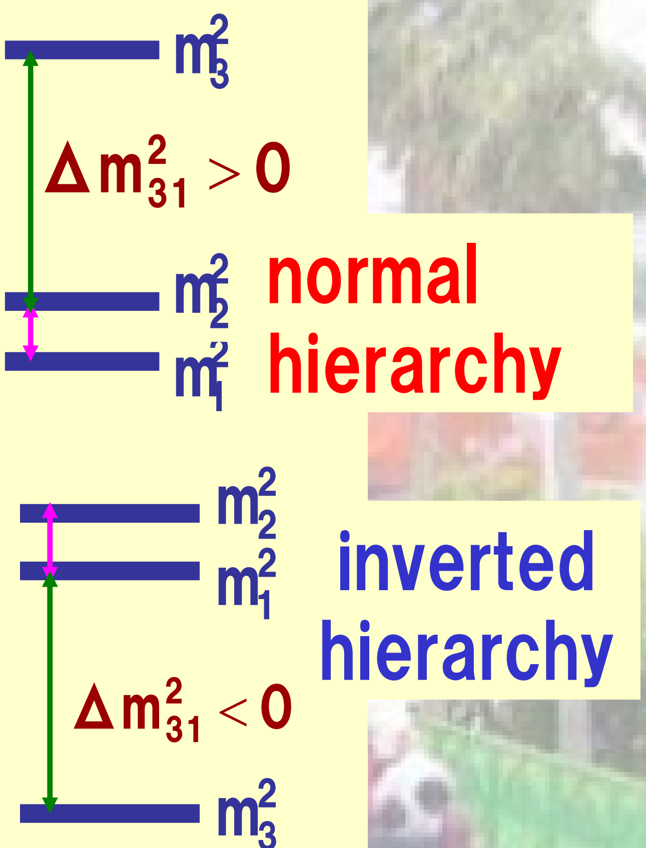
$$P \equiv P(\nu_\mu \to \nu_e)$$

$$\bar{P} \equiv P(\overline{\nu_\mu} \to \overline{\nu_e})$$

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E}$$

Oscillation Maximum (OM)

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E} = \frac{\pi}{2}$$



Plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) plane

The way curves intersect is easy to see

(P=const, δ=const)

($\bar{P}$=const, δ=const)

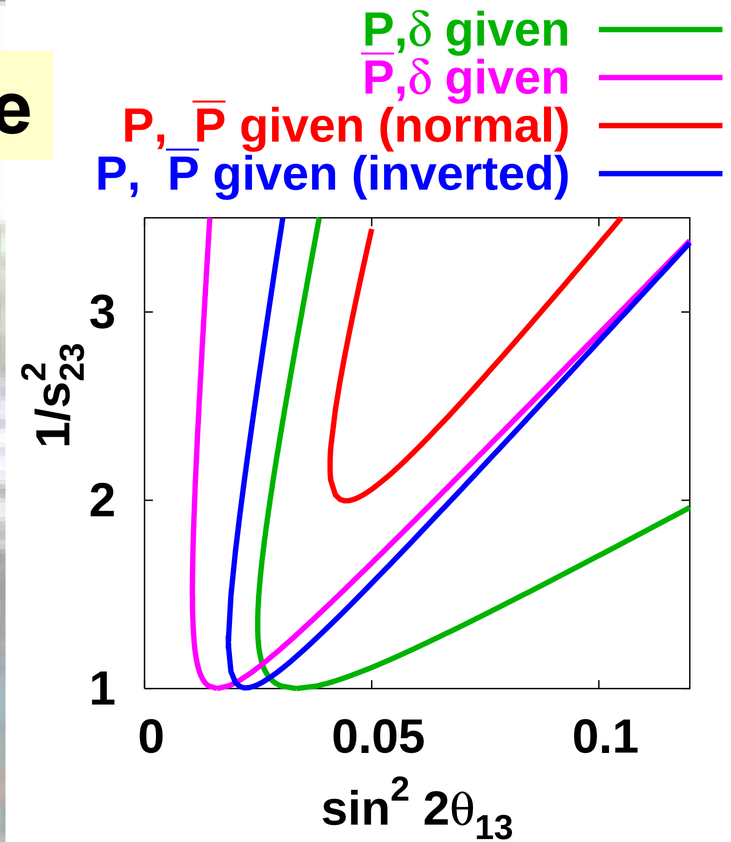
(P=const& $\bar{P}$=const' **off OM**)

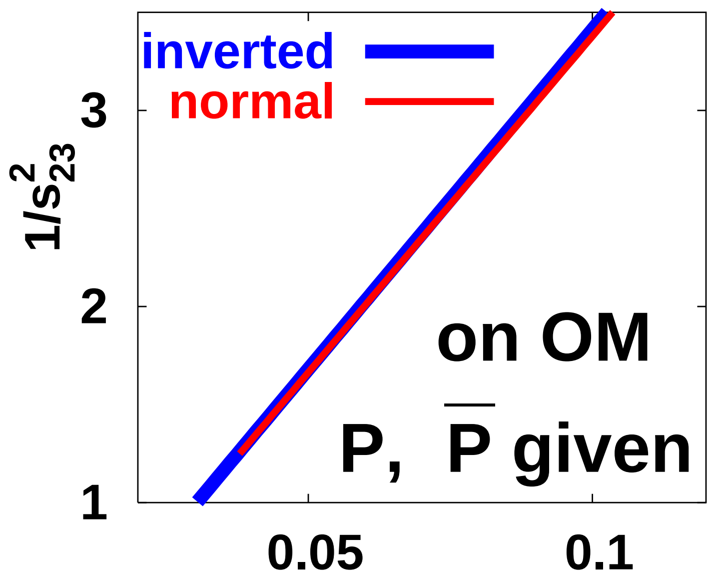
hyperbolas

(or ellipses)

(P=const& $\bar{P}$=const' **on OM**)

straight lines





2. $|U_{e3}|=s_{13}$

Assumption: at JPARC (@OM, 4MW, HK)

$\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured.

Question: Will that be enough to determine $|U_{e3}|$?

Answer: It depends on

(1) $\sin^2 2\theta_{23} \cong 1$

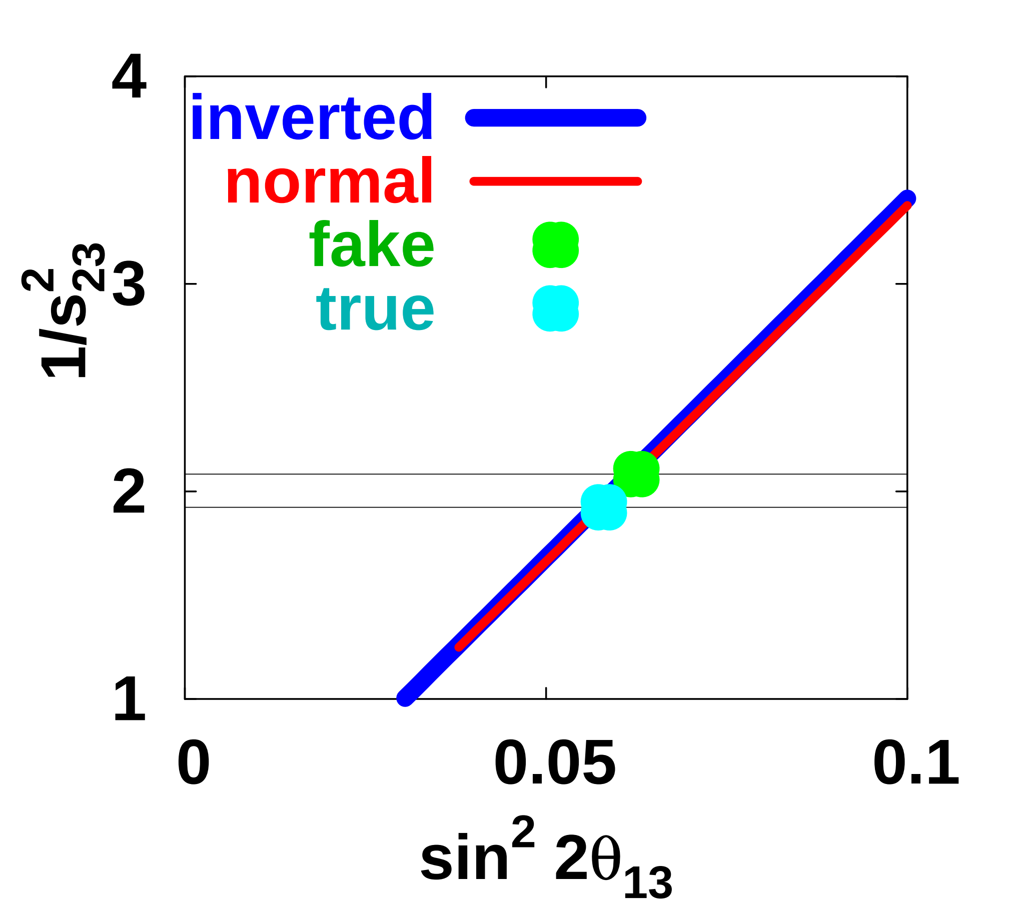
(2) $\sin^2 2\theta_{23} < 1$

(1) $\sin^2 2\theta_{23} \simeq 1$

JPARC $\nu + \bar{\nu}$ is almost enough, since

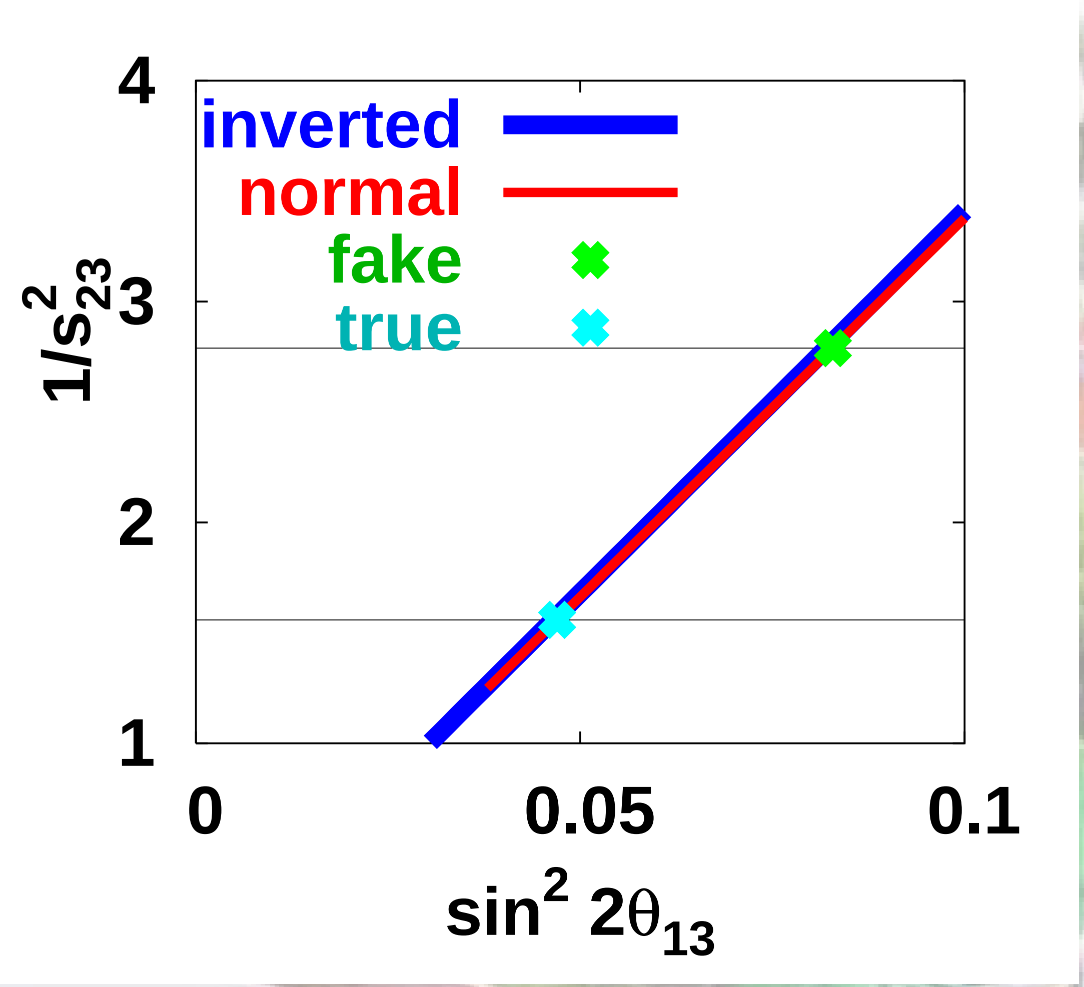
(a) there is no intrinsic (δ, θ_{13}) degeneracy

(b) $\mathrm{sign}(\Delta m^2_{31})$ degeneracy is small



(2) $\sin^2 2\theta_{23} < 1$

Ambiguity due to $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy is significant.



To resolve θ_{23} ambiguity, possible ways are:

(A) reactor measurement of θ_{13}

O.Y.@NOON2003

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

Parke @NOON2003

(C) measurement of $\nu_e \to \nu_\tau$

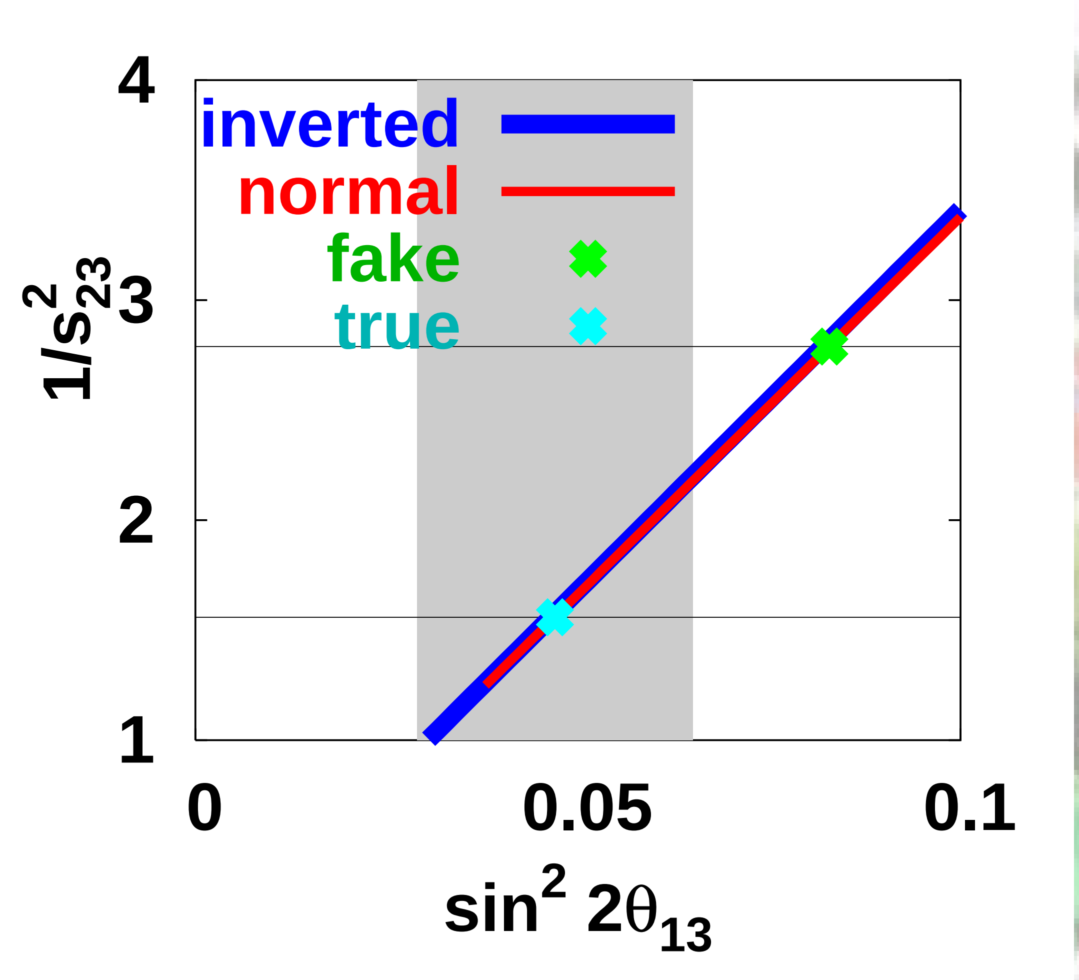
Donini@NOON2003

The reference values used here are:

$\sin^2 2\theta_{23}=0.96$, $\sin^2 2\theta_{13}=0.05$, $\delta=\pi/4$, $\Delta m^2_{31}>0$

(A) reactor measurement of θ 13

One can resolve θ 23 ambiguity at 90%CL.

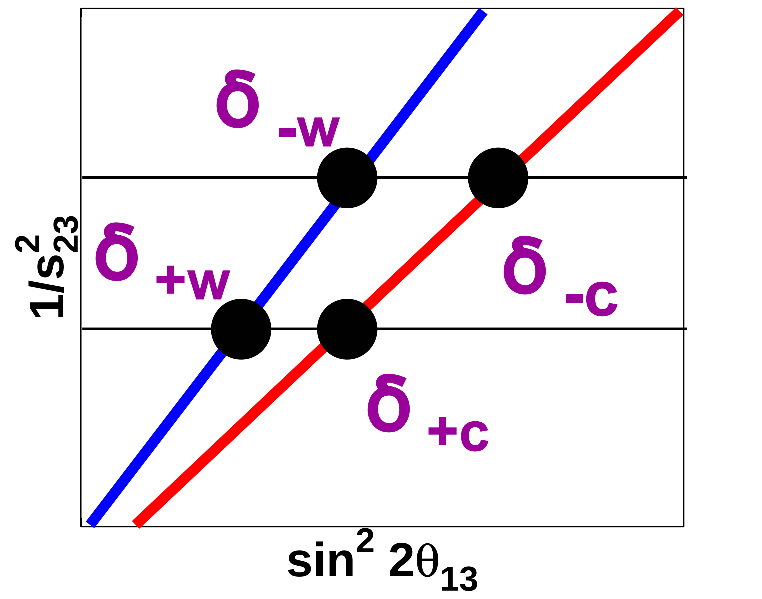


(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

Consider 3rd measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

in addition to JPARC $\nu + \bar{\nu}$.

↓ (exaggerated figure)



correct assumption
wrong assumption
on mass hierarchy

The value of δ for each point can be deduced (up to δ ⇔ π - δ) from

$$\sin\delta = -\frac{P - f^2x^2 - g^2y^2}{2fgxy},$$

$$x \equiv s_{23}\sin 2\theta_{13},$$

$$y \equiv |\Delta m^2_{21}/\Delta m^2_{31}| c_{23}\sin 2\theta_{12},$$

$$f, \bar{f} \equiv \sin(\Delta \mp AL/2)/(1 \mp AL/2\Delta),$$

$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

Then from the equation for the probability of $\nu_\mu \rightarrow \nu_e$ (or $\nu_e \rightarrow \nu_\mu$) in the 3rd experiment

$$P_{true} = P\left(\sin^2 2\theta_{13}, \delta_{\pm[cw]}, s^2_{23} \right)$$

or

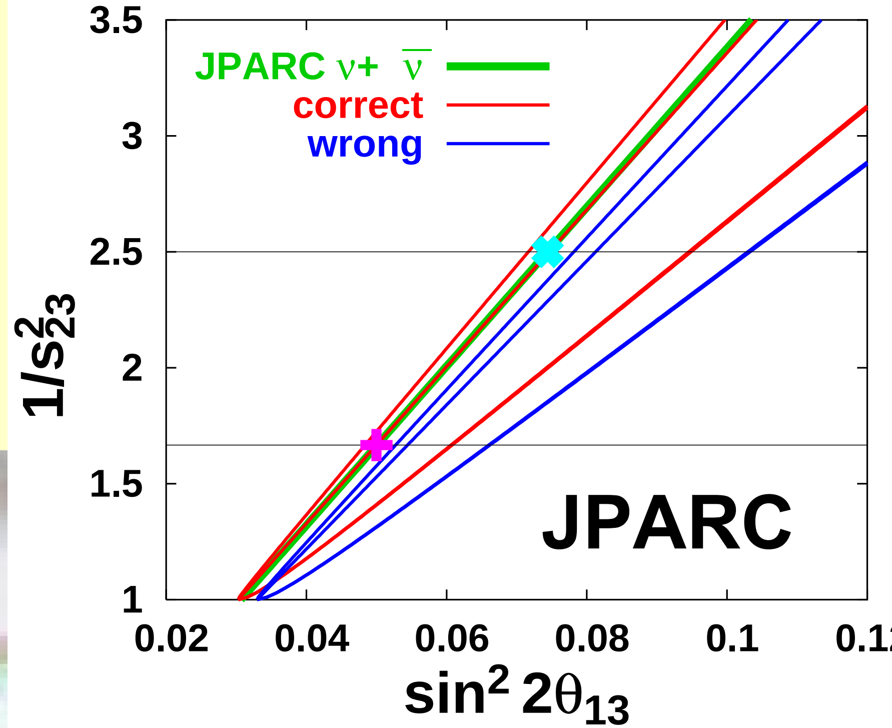
$$P_{true} = P\left(\sin^2 2\theta_{13}, \pi - \delta_{\pm[cw]}, s^2_{23} \right)$$

where

$$P_{true} \equiv P\left((\sin^2 2\theta_{13})_{true}, \delta_{true}, (s^2_{23})_{true} \right)$$

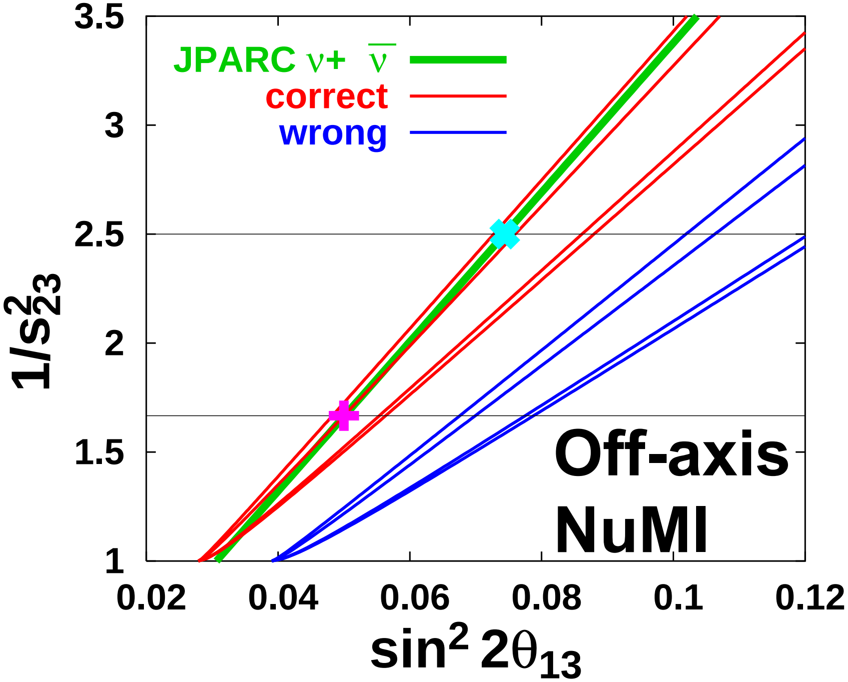
we can get a unique line (a hyperbola or an ellipse) in $(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane for $\delta_{\pm[cw]}$ or $\pi - \delta_{\pm[cw]}$.

L = 295 km, E=1.19 GeV, P=0.0158

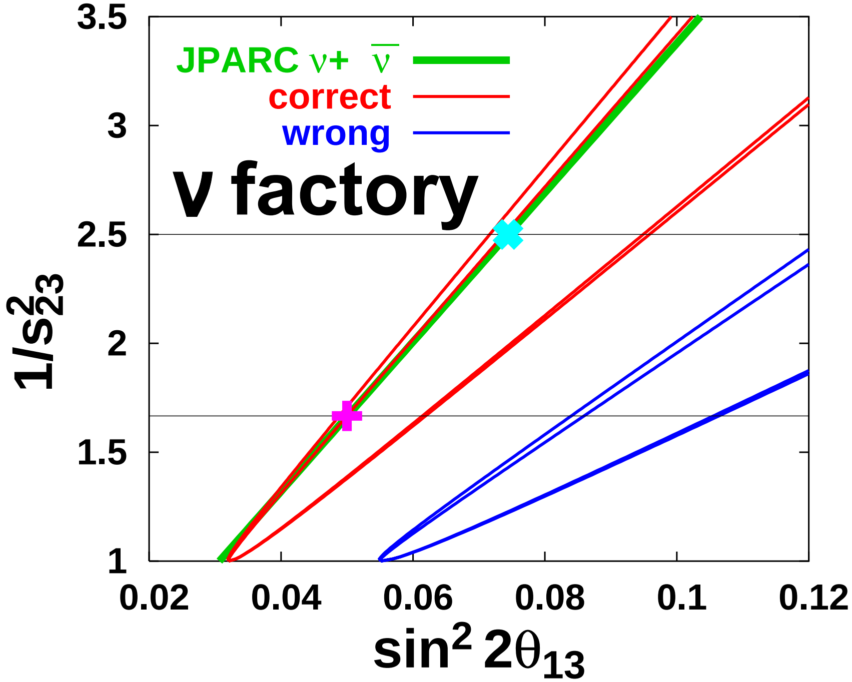


In general, the gradient of the hyperbola is almost equal to that of the JPARC line, and this additional curve does not help to resolve θ_{23} ambiguity if $\Delta \leqq \pi/2$.

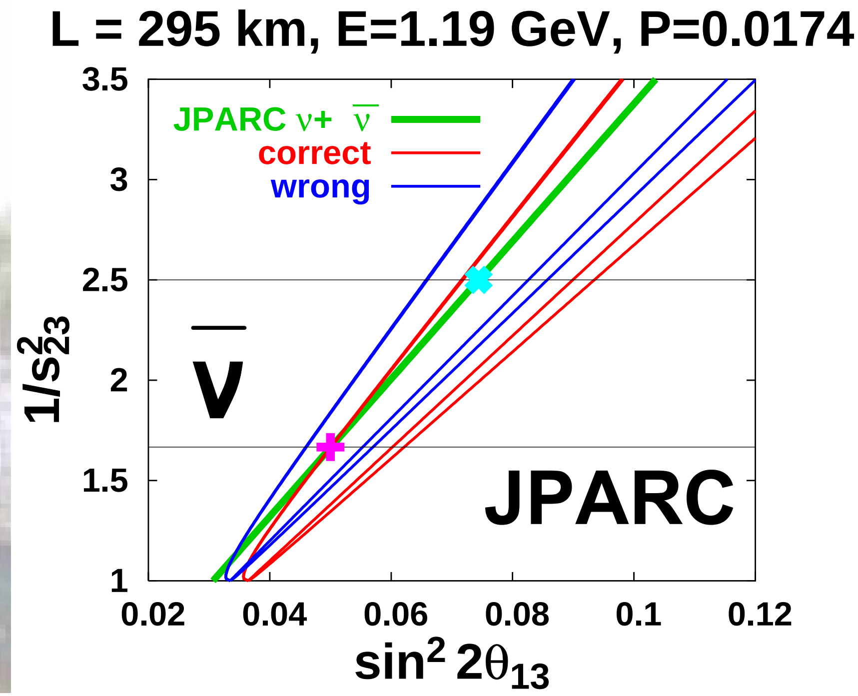
L = 730 km, E=1.97 GeV, P=0.0277

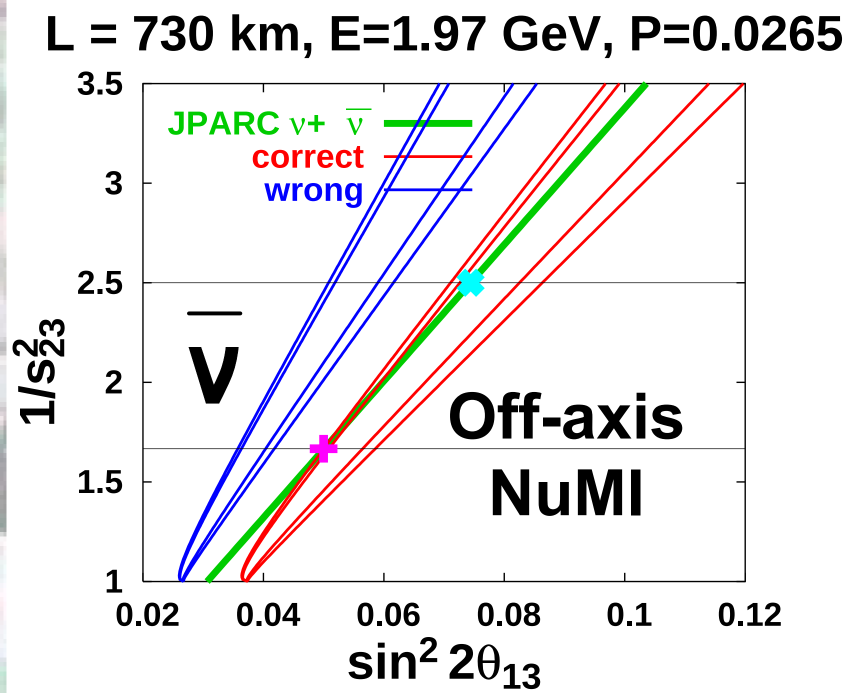


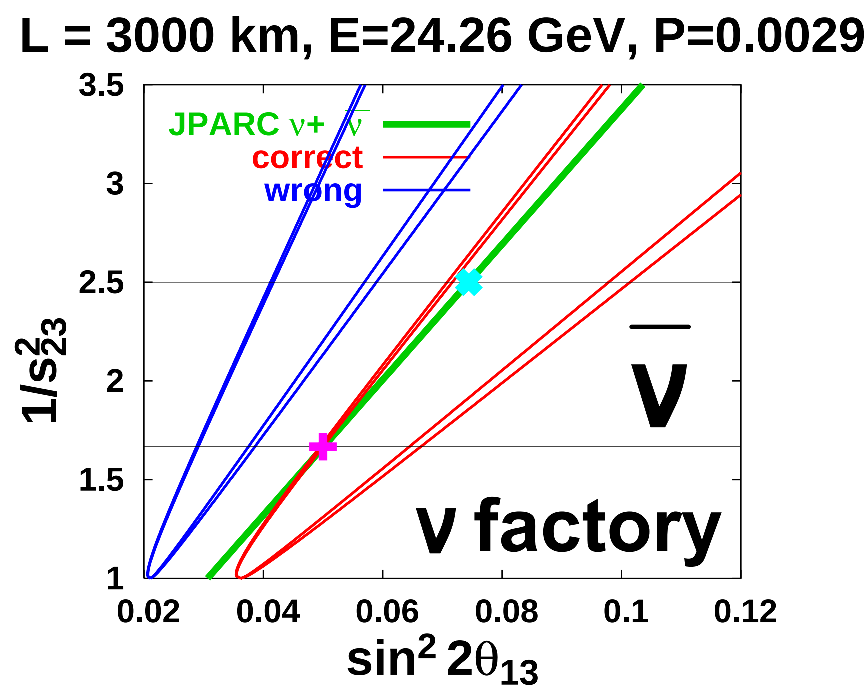
L = 3000 km, E=24.26 GeV, P=0.0044



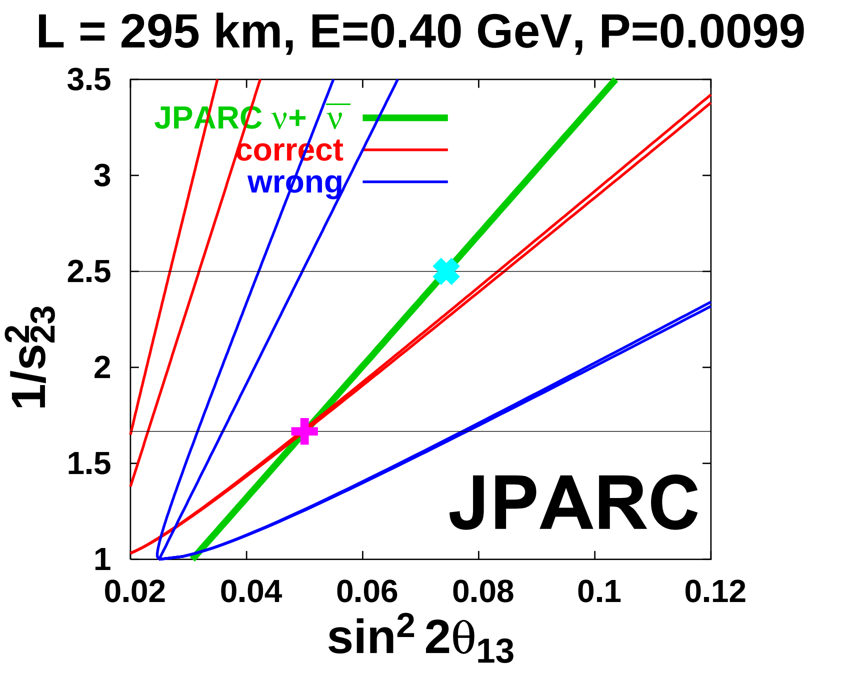
The situation doesn't change much for $\bar{\nu}_\mu \to \bar{\nu}_e$ if $\Delta \leqq \pi/2$.







On the other hand, for $\pi/2 < \Delta < \pi$, the situation is different.



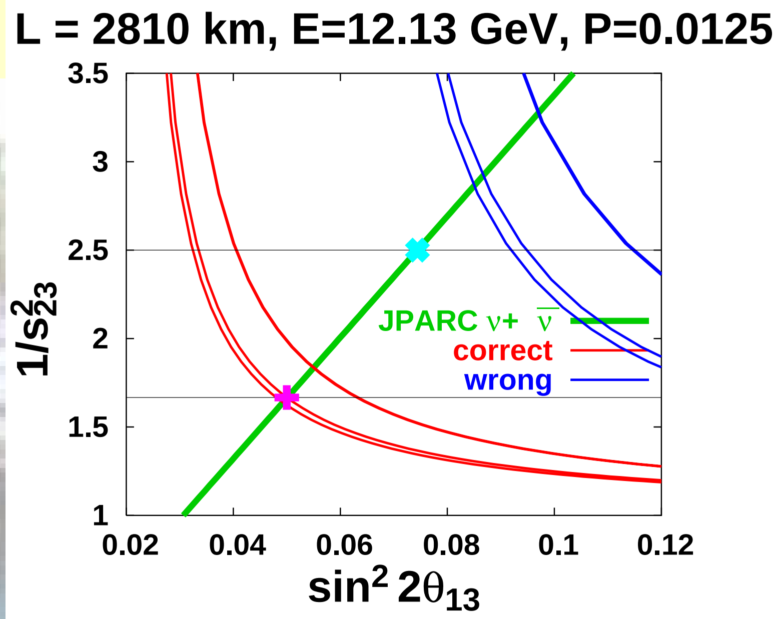
Good news is

- θ_{23} ambiguity may be resolved.
- $\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.

Bad news is

- E is so low that statistics is low.
- osc. prob. Is small (∼solar ν osc. prob.).

(C) measurement of $\nu_e \to \nu_\tau$



Curves intersect with the JPARC line almost orthogonally.

- **θ_{23} ambiguity may be resolved.**
- **$\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.**
- **sign(Δm^2_{31}) ambiguity may be resolved .**

This channel may be interesting to be combined with JPARC in the future.

3. $\arg(U_{e3}) = \delta$

Assumption: at JPARC (@OM, 4MW, HK)

$\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured.

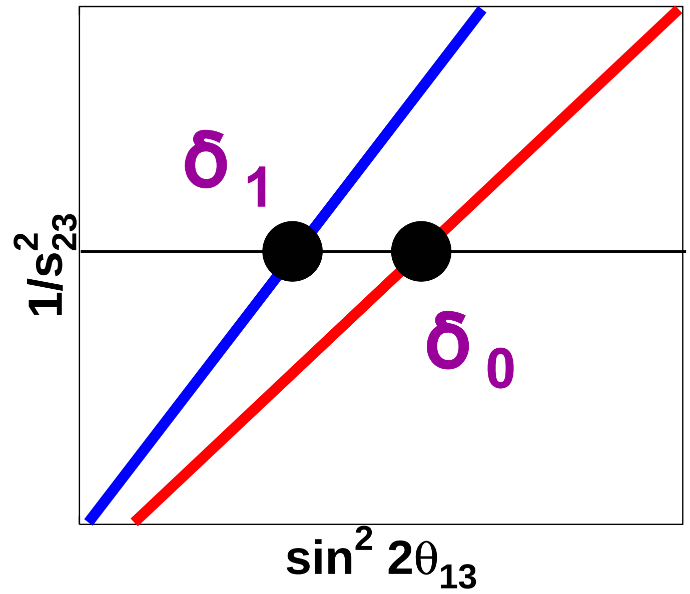
Question:

Will that be enough to determine $\arg(U_{e3})$?

Answer: In general no.

Resolution of sign(Δm^2_{31}) ambiguity is important.

(1) $\sin^2 2\theta_{23} \simeq 1$



δ_0 : by correct assumption =true value

δ_1 : by wrong assumption on sign(Δm^2_{31})

Difference between δ_0 & δ_1 turns out to be large.

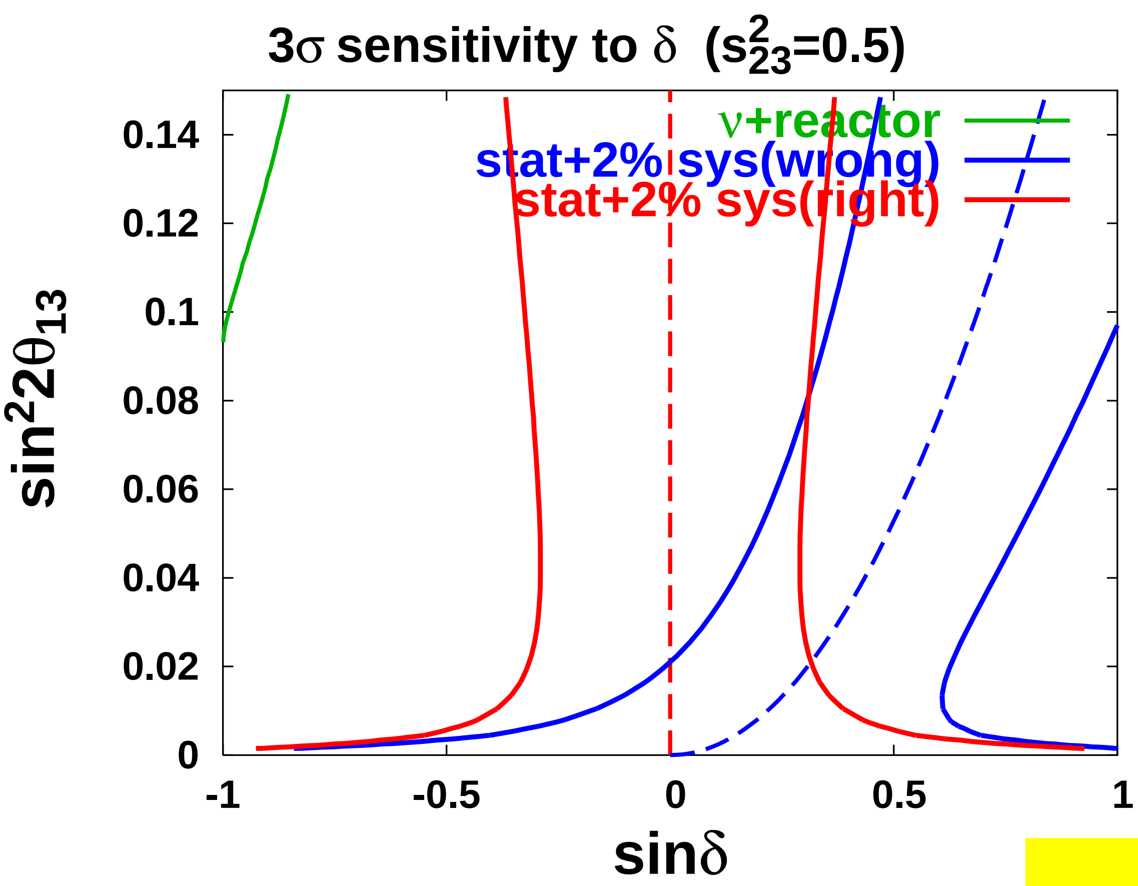
If δ_0 =0, then $\sin\delta_1 \simeq -2.2 \sin 2\theta_{13}$ at JPARC

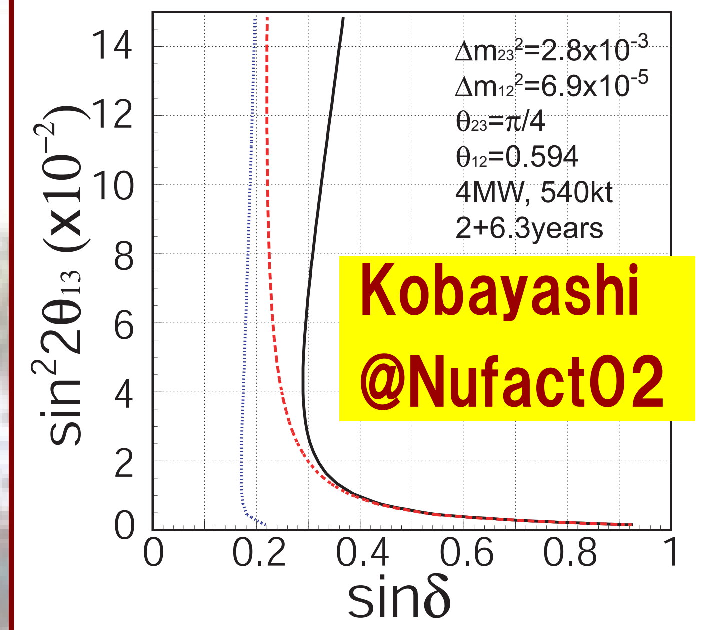
= -0.5 (if $\sin^2 2\theta_{13}$=0.05)

Identification of sign(Δm^2_{31}) is important.

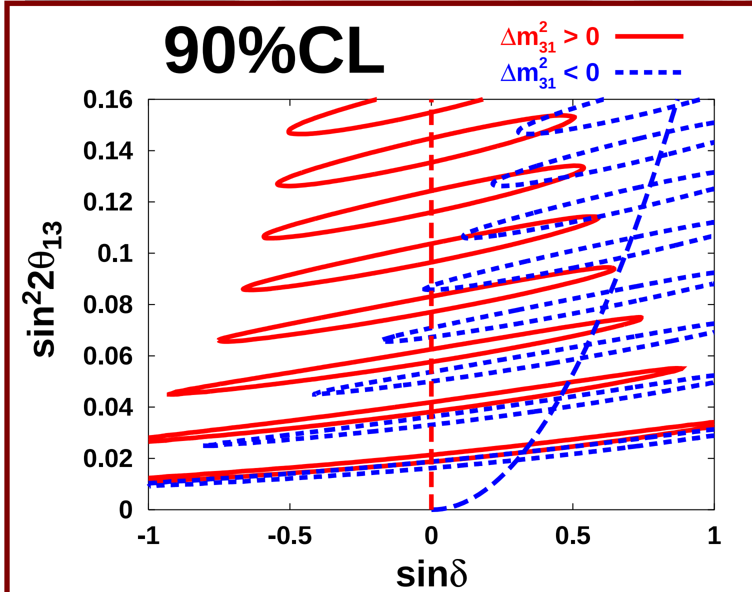
3σ sensitivity to δ

Assuming $\Delta m^2_{31}>0$



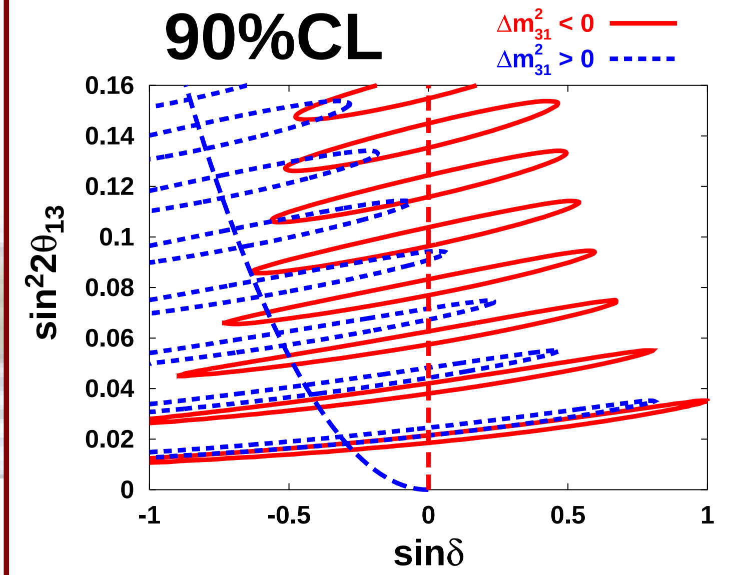


Kobayashi
@Nufact02

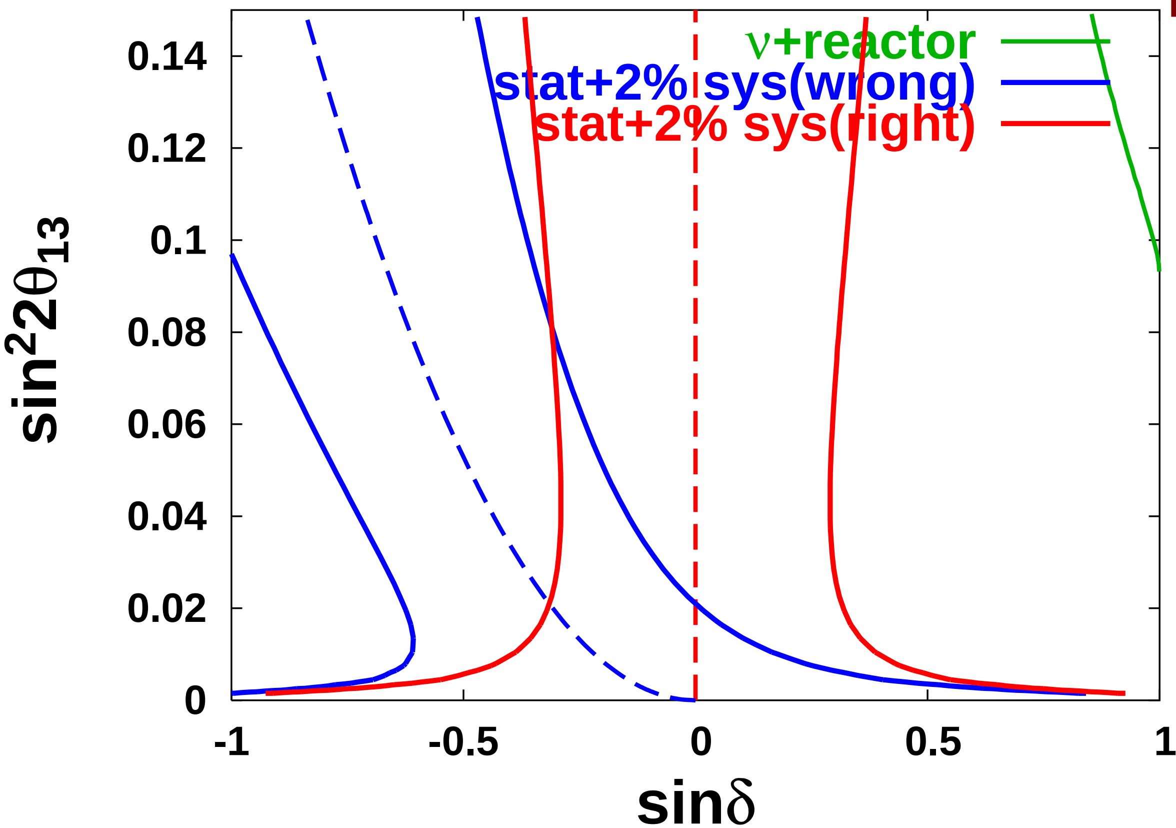


↑modified from
Minakata-Sugiyama (PLB580,216)

Assuming $\Delta m^2_{31}<0$

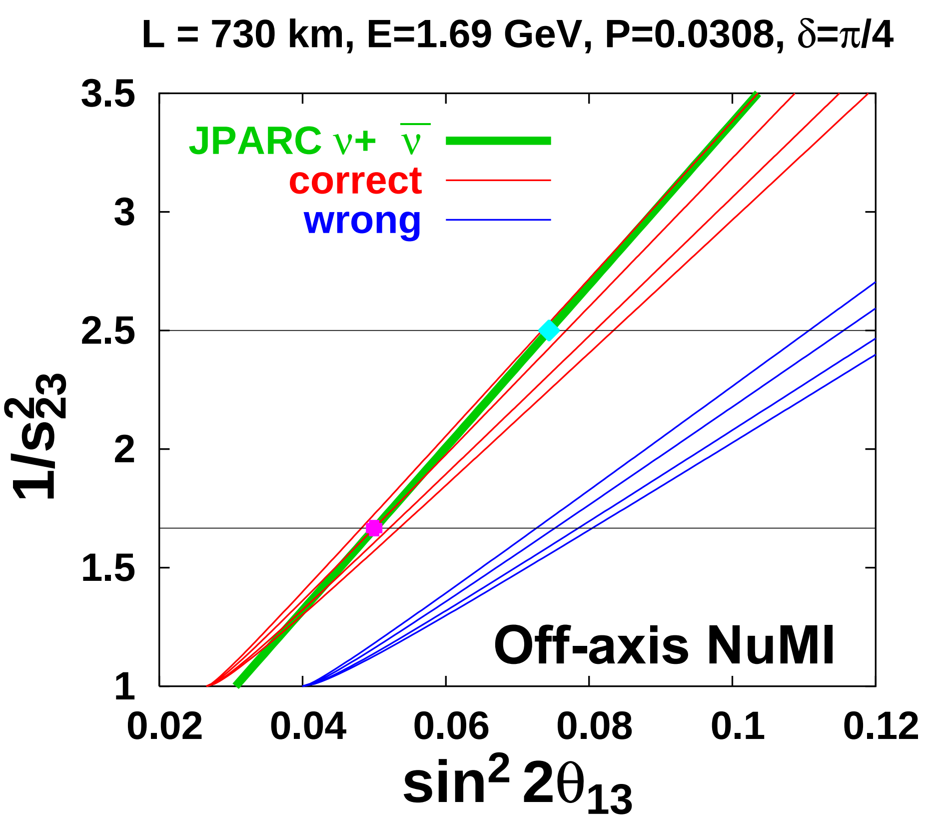


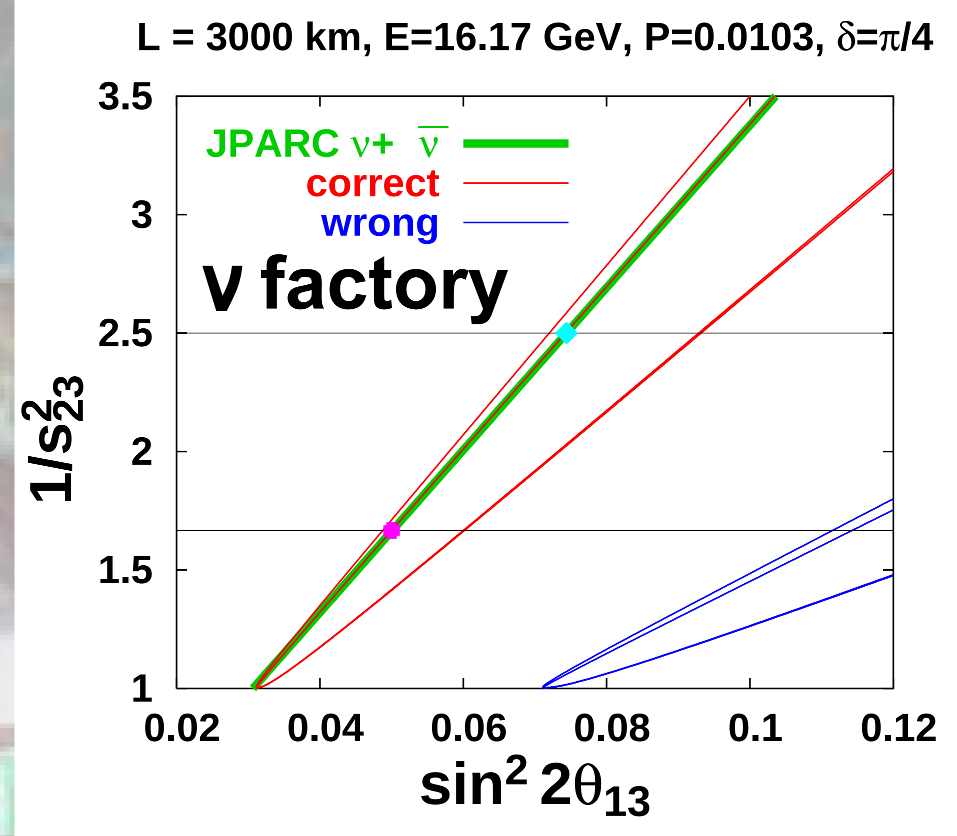
3σ sensitivity to δ (s^2_{23}=0.5)



↑modified from Minakata-Sugiyama (PLB580,216)

LBL experiments with longer baselines are advantageous to resolve sign(Δm^2_{31}) ambiguity.



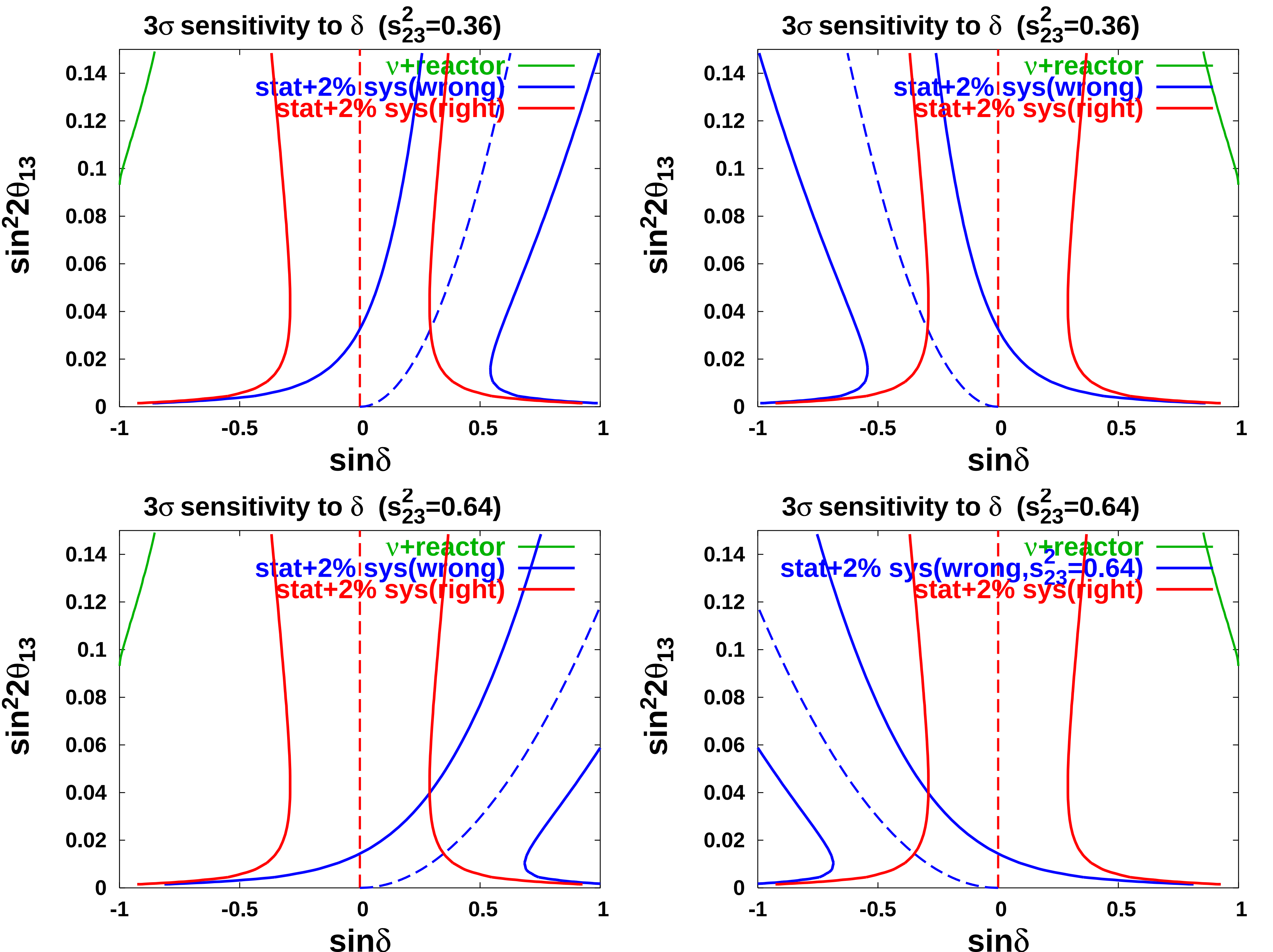


(2) $\sin^2 2\theta_{23} < 1$

Ambiguity of δ due to θ_{23} ambiguity:

$$\sin\delta_2 \simeq \frac{1}{t_{23}} \sin\delta_0$$

- **If $\sin\delta$ and $|\cos 2\theta_{23}|$ are both large, then this ambiguity has to be taken into account.**



3σ sensitivity to δ (s^2_{23}=0.36)
ν+reactor
stat+2% sys(wrong)
stat+2% sys(right)
$\sin^2 2\theta_{13}$
sinδ
3σ sensitivity to δ (s^2_{23}=0.36)
ν+reactor
stat+2% sys(wrong)
stat+2% sys(right)
$\sin^2 2\theta_{13}$
sinδ
3σ sensitivity to δ (s^2_{23}=0.64)
ν+reactor
stat+2% sys(wrong)
stat+2% sys(right)
$\sin^2 2\theta_{13}$
sinδ
3σ sensitivity to δ (s^2_{23}=0.64)
ν+reactor
stat+2% sys(wrong,s^2_{23}=0.64)
stat+2% sys(right)
$\sin^2 2\theta_{13}$
sinδ

4. Summary

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E}$$

△:may be OK

		intrinsic	$\delta \leftrightarrow \pi-\delta$	$\mathrm{sign}(\Delta m^2)$	θ_{23} $\left(\text{if } \theta_{23} \neq \frac{\pi}{4}\right)$
JPARC $\nu+\bar{\nu}$		✓	×	×	×
JPARC $\nu+\bar{\nu}$ + reactor (90%CL)		✓	×	×	✓
JPARC $\nu+\bar{\nu}$ + LBL (ν and/or $\bar{\nu}$)	$\Delta<\pi/8$	✓	✓	×	×
	$\Delta<\pi/2$, L<500km	✓	△	×	△
	$\Delta<\pi/2$, L>500km	✓	△	✓	△
	$\Delta>\pi/2$	✓	△	△	△
JPARC $\nu+\bar{\nu}$ + $\nu_e \rightarrow \nu_\tau$	L<500km	✓	△	×	✓
	L>500km	✓	△	✓	✓

It is important

- for determination of θ_{13}

  to resolve θ_{23} ambiguity

  if $\sin^2 2\theta_{23} < 1$.

- for determination of δ

  to resolve sign(Δm^2_{31}) ambiguity.

Appendices

$$Y \equiv 1/s_{23}^2, \quad X \equiv \sin^2 2\theta_{13}, \quad C \equiv \alpha^2 g^2 \sin^2 2\theta_{12}$$

$\nu + \bar{\nu}$ @OM

$$Y_{normal} = \frac{f + \bar{f}}{P/f + \bar{P}/\bar{f}} \left(X - \frac{C}{f\bar{f}} \right)$$

$$Y_{inverted} = \frac{f + \bar{f}}{P/\bar{f} + P/\bar{f}} \left(X - \frac{C}{f\bar{f}} \right)$$

$$Y \equiv 1/s_{23}^2, \quad X \equiv \sin^2 2\theta_{13}, \quad C \equiv \alpha^2 g^2 \sin^2 2\theta_{12}, \quad \lambda \equiv C/P$$

$$f, \bar{f} \equiv \sin\left(\Delta \mp AL/2\right)/\left(1 \mp AL/2\Delta\right), \quad g \equiv \sin\left(AL/2\right)/\left(AL/2\Delta\right)$$

ν only (normal hierarchy)

$$\frac{f^2}{P} X = 1 + \left[1 + \frac{2\lambda}{1-\lambda}\cos^2(\delta+\Delta)\right]$$
$$-\cos(\delta+\Delta)\sqrt{4\lambda(Y-1)\left[\left(1-\lambda+\lambda\cos^2(\delta+\Delta)\right)(Y-1)+1\right]}$$

ν only (inverted hierarchy)

$$\frac{\bar{f}^2}{P} X = 1 + \left[1 + \frac{2\lambda}{1-\lambda}\cos^2(\delta+\Delta)\right]$$
$$-\cos(\delta+\Delta)\sqrt{4\lambda(Y-1)\left[\left(1-\lambda+\lambda\cos^2(\delta+\Delta)\right)(Y-1)+1\right]}$$

normal hierarchy

$$P = f^2 x^2 + 2xyfg\cos(\delta + \Delta) + g^2 y^2$$

$$\bar{P} = \bar{f}^2 x^2 + 2xy\bar{f}g\cos(\delta - \Delta) + g^2 y^2$$

inverted hierarchy

$$P = \bar{f}^2 x^2 - 2xy\bar{f}g\cos(\delta + \Delta) + g^2 y^2$$

$$\bar{P} = f^2 x^2 - 2xyfg\cos(\delta - \Delta) + g^2 y^2$$

$$\Delta \equiv \frac{|\Delta m^2_{31}|L}{4E}$$

$$x \equiv s_{23}\sin 2\theta_{13},$$

$$y \equiv |\Delta m^2_{21}/\Delta m^2_{31}|\, c_{23}\sin 2\theta_{12},$$

$$f, \bar{f} \equiv \sin(\Delta \mp AL/2)/(1 \mp AL/2\Delta),$$

$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

A. Mass hierarchy degeneracy: sgn(δm^2_{31}) ambiguity

$$x'^2 = \frac{x^2(f^2+\bar{f}^2 - f\bar{f}) - 2yg(f-\bar{f})x\sin\delta\sin\Delta}{f\bar{f}},$$

$$x'\sin\delta' = x\sin\delta\frac{f^2+\bar{f}^2-f\bar{f}}{f\bar{f}} - \frac{x^2}{\sin\Delta}\frac{f^2+\bar{f}^2}{f\bar{f}}\frac{f-\bar{f}}{2yg}.$$

en Eq. (2) reduces to

$$\sin\delta' = -x\frac{f^2+\bar{f}^2}{f\bar{f}}\frac{f-\bar{f}}{2yg\sin\Delta}\sqrt{\frac{f\bar{f}}{f^2+\bar{f}^2-f\bar{f}}},$$

Barger Marfatia Whisnant

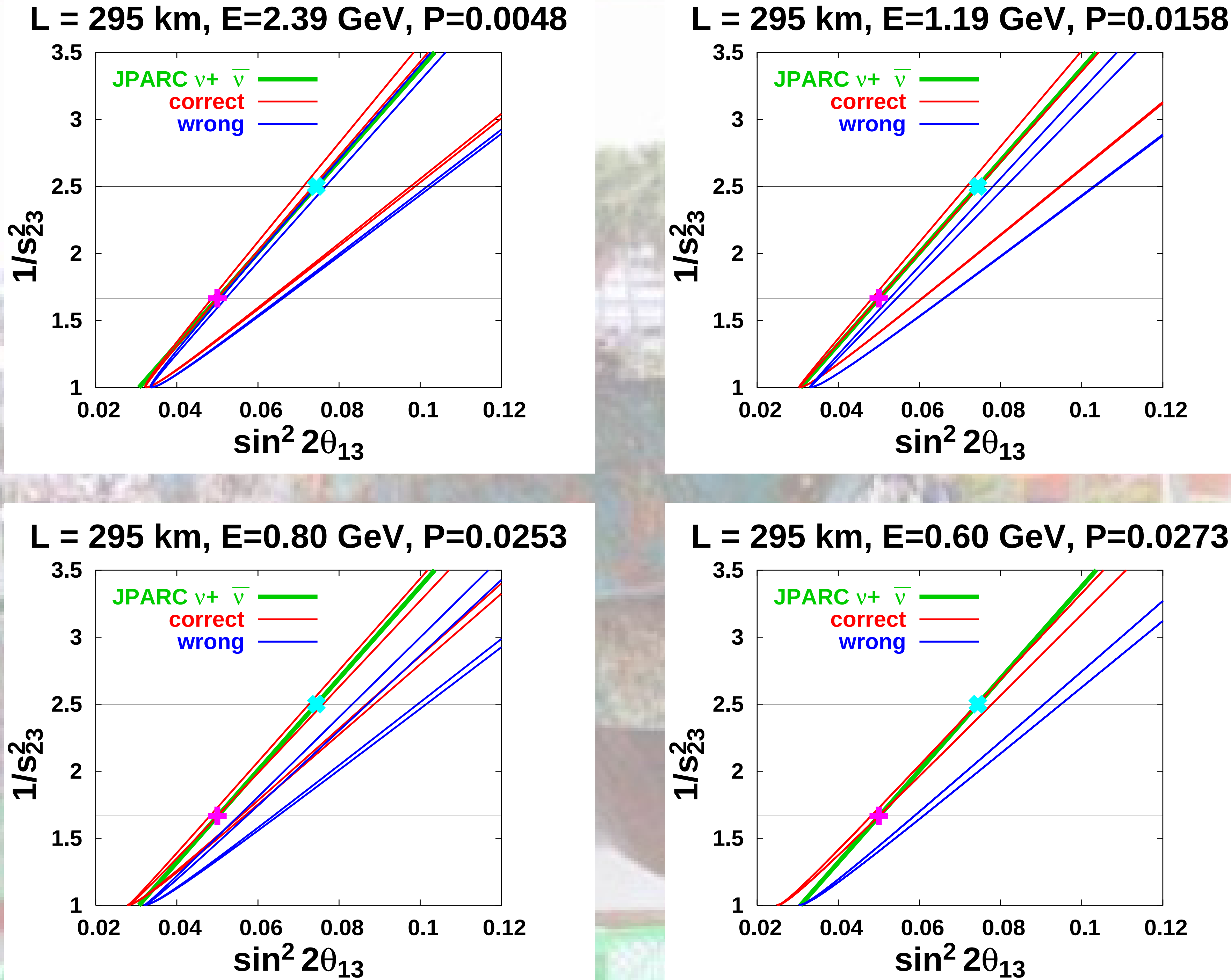
Phys.Rev.D65:073023,2002

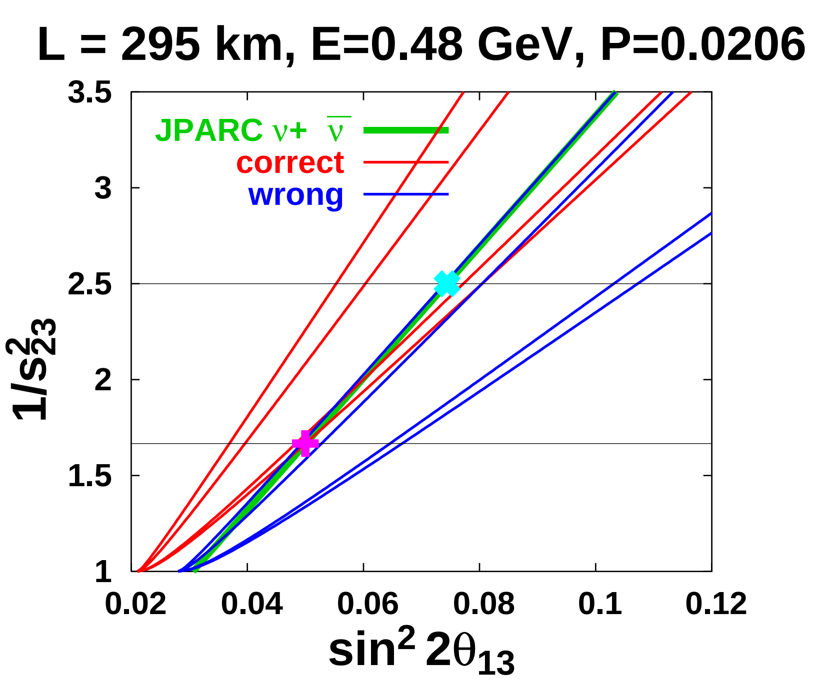
B. Atmospheric angle degeneracy: $(\theta_{23}\,,\pi/2-\theta_{23})$ ambiguity

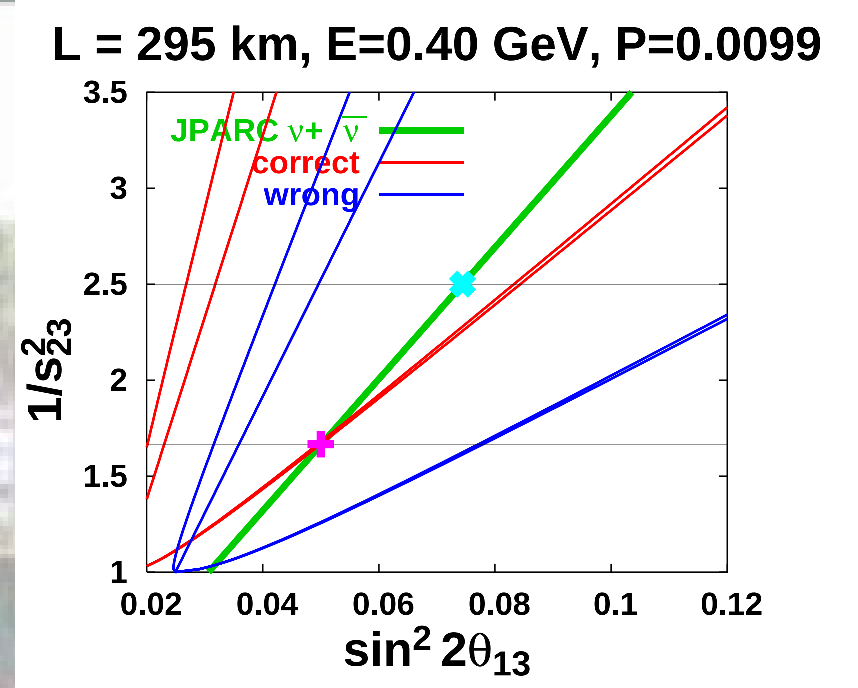
$$\sin^2 2\theta'_{13} = \sin^2 2\theta_{13}\tan^2\theta_{23} + \frac{\alpha^2 g^2 \sin^2 2\theta_{12}}{f\bar{f}}(1-\tan^2\theta_{23})\,,$$

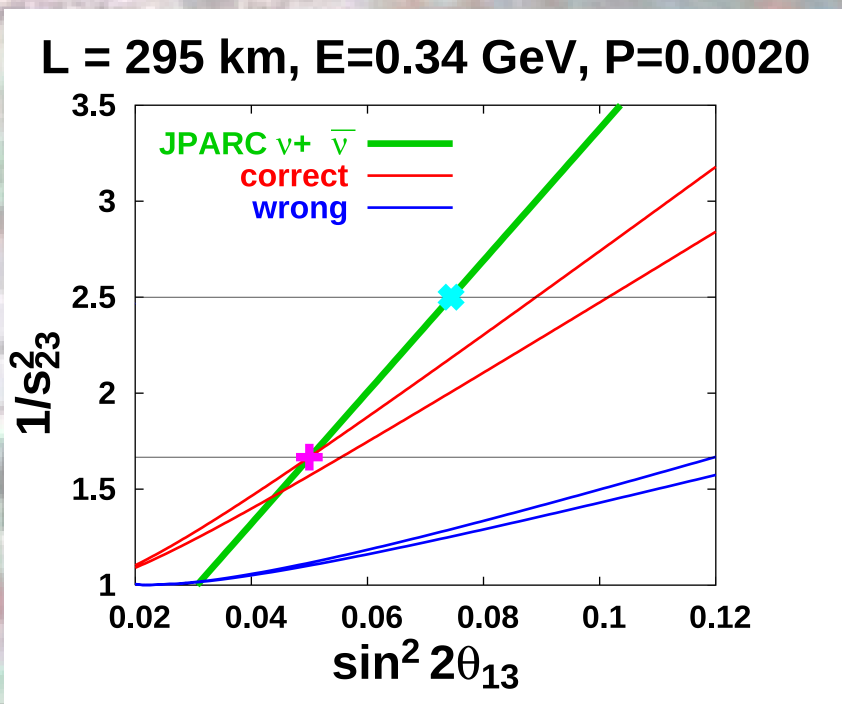
$$\sin 2\theta'_{13}\sin\delta' = \sin 2\theta_{13}\sin\delta + \frac{\alpha g(f-\bar{f})\sin 2\theta_{12}}{f\bar{f}}\frac{\cot 2\theta_{23}}{\sin\Delta}\,,$$

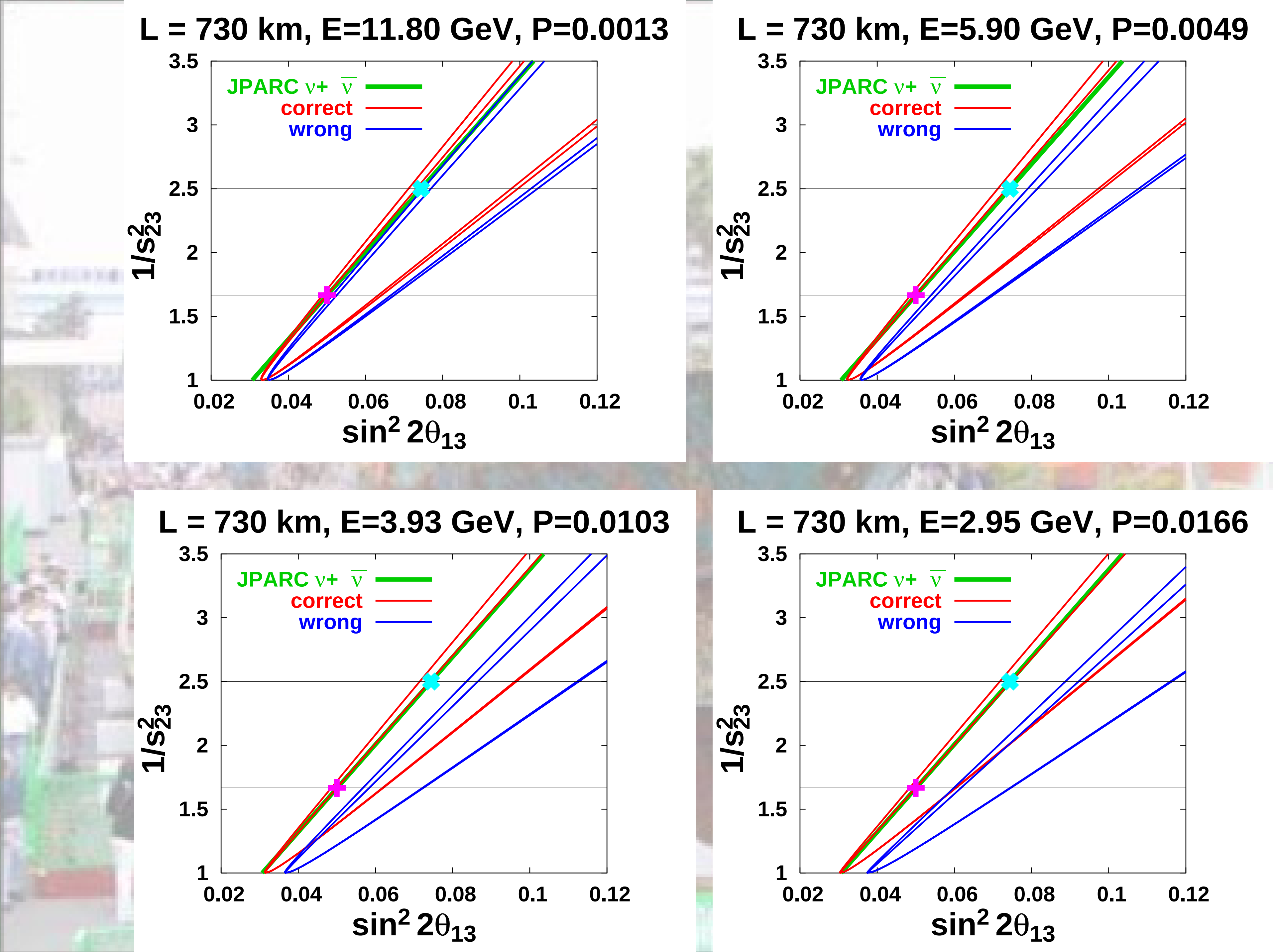
Barger Marfatia Whisnant
Phys.Rev.D65:073023,2002


L = 295 km, E=2.39 GeV, P=0.0048
L = 295 km, E=1.19 GeV, P=0.0158
L = 295 km, E=0.80 GeV, P=0.0253
L = 295 km, E=0.60 GeV, P=0.0273
JPARC ν+ ν̄
correct
wrong
$1/s^2_{23}$
$\sin^2 2\theta_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

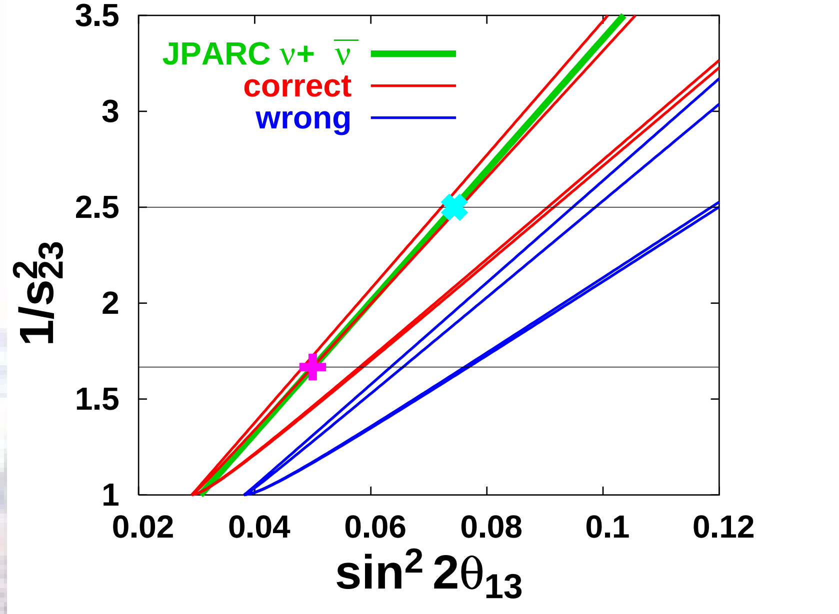

L = 295 km, E=0.48 GeV, P=0.0206
JPARC ν+ ν̄
correct
wrong
1/s$^2_{23}$
sin^2 2θ$_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12


L = 295 km, E=0.40 GeV, P=0.0099
JPARC ν+ ν̄
correct
wrong
1/s$^2_{23}$
sin^2 2θ$_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

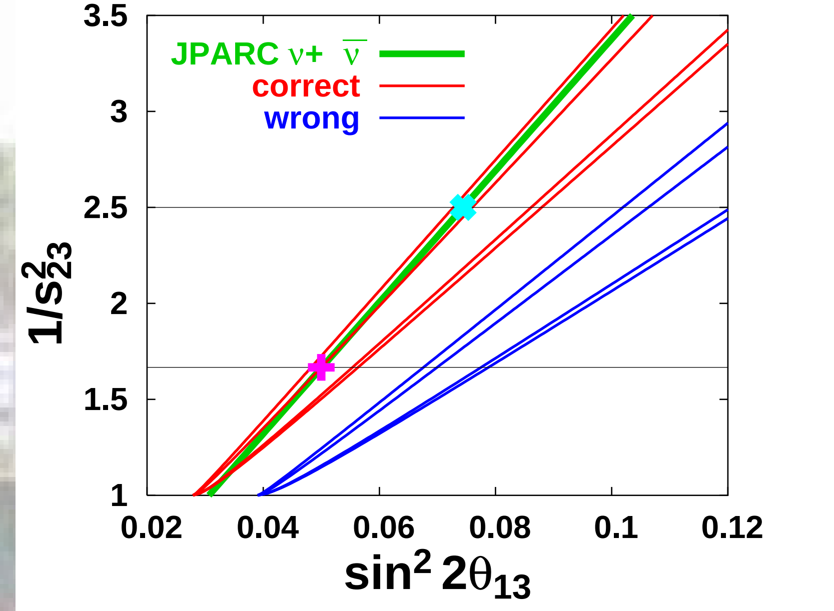

L = 295 km, E=0.34 GeV, P=0.0020
JPARC ν+ ν̄
correct
wrong
1/s$^2_{23}$
sin^2 2θ$_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12


L = 730 km, E=11.80 GeV, P=0.0013
L = 730 km, E=5.90 GeV, P=0.0049
L = 730 km, E=3.93 GeV, P=0.0103
L = 730 km, E=2.95 GeV, P=0.0166
JPARC ν+ ν̄
correct
wrong
$1/s^2_{23}$
$\sin^2 2\theta_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

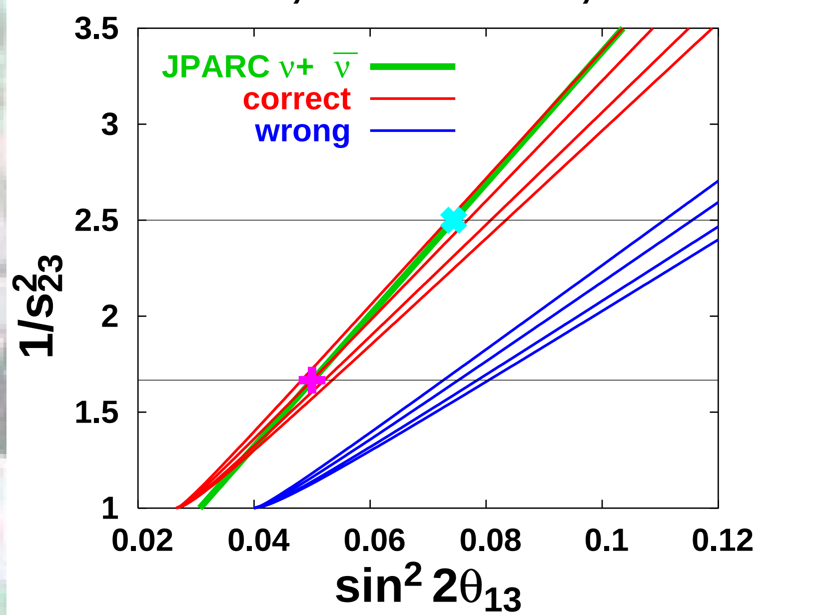
L = 730 km, E=2.36 GeV, P=0.0227

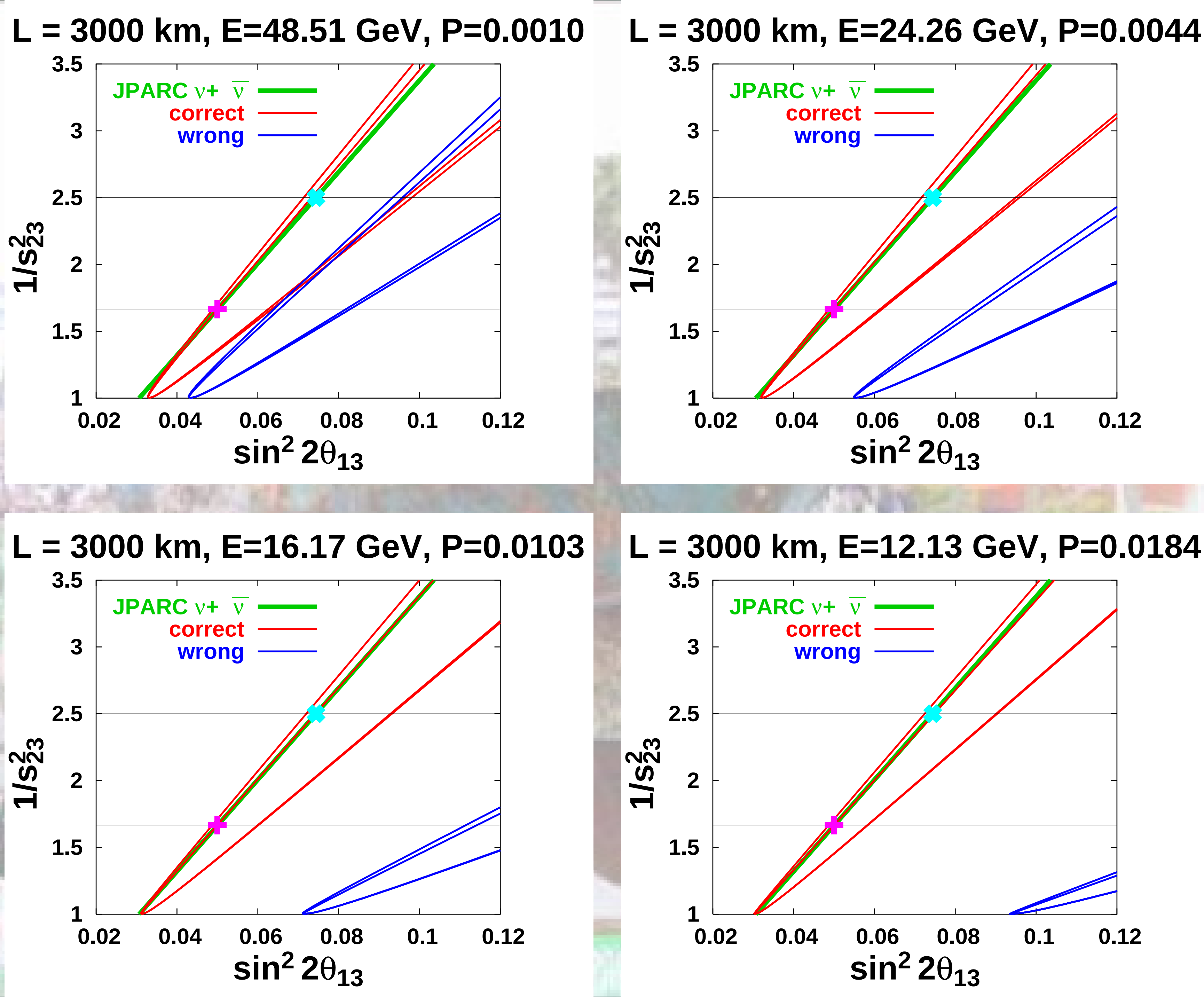


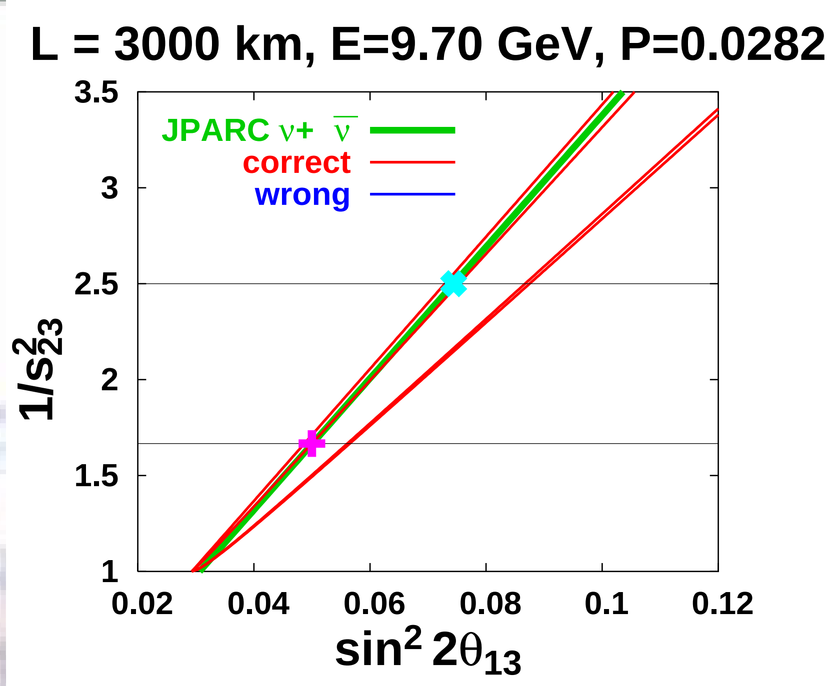
L = 730 km, E=1.97 GeV, P=0.0277

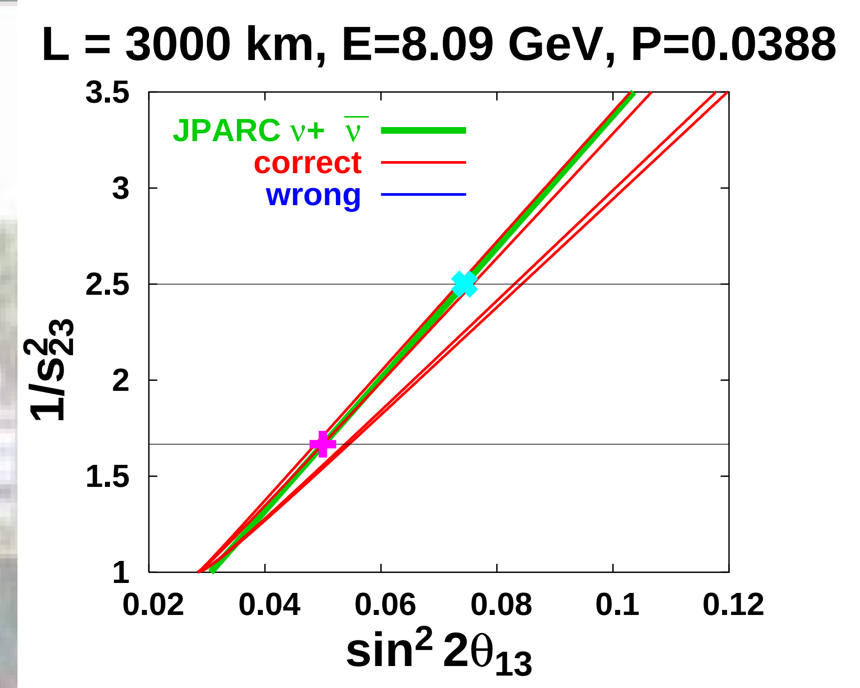


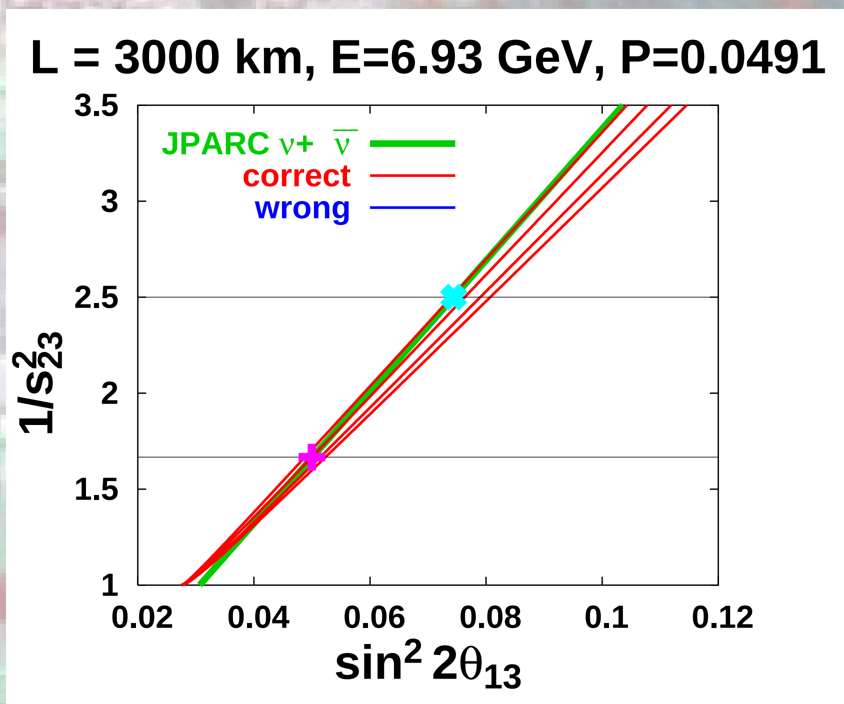
L = 730 km, E=1.69 GeV, P=0.0308

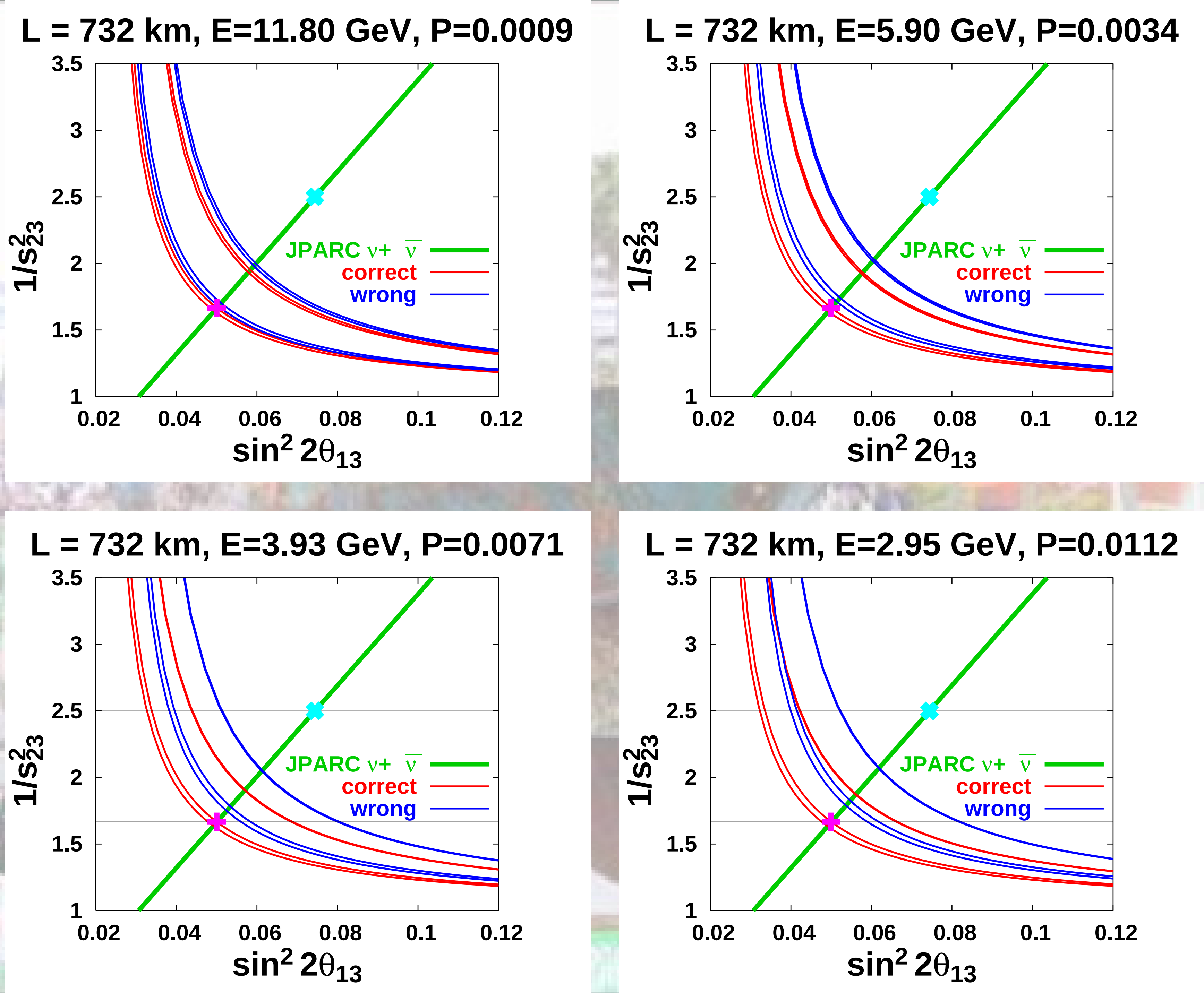



L = 3000 km, E=48.51 GeV, P=0.0010
L = 3000 km, E=24.26 GeV, P=0.0044
L = 3000 km, E=16.17 GeV, P=0.0103
L = 3000 km, E=12.13 GeV, P=0.0184
JPARC ν+ $\overline{\nu}$
correct
wrong
$1/s^2_{23}$
$\sin^2 2\theta_{13}$
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

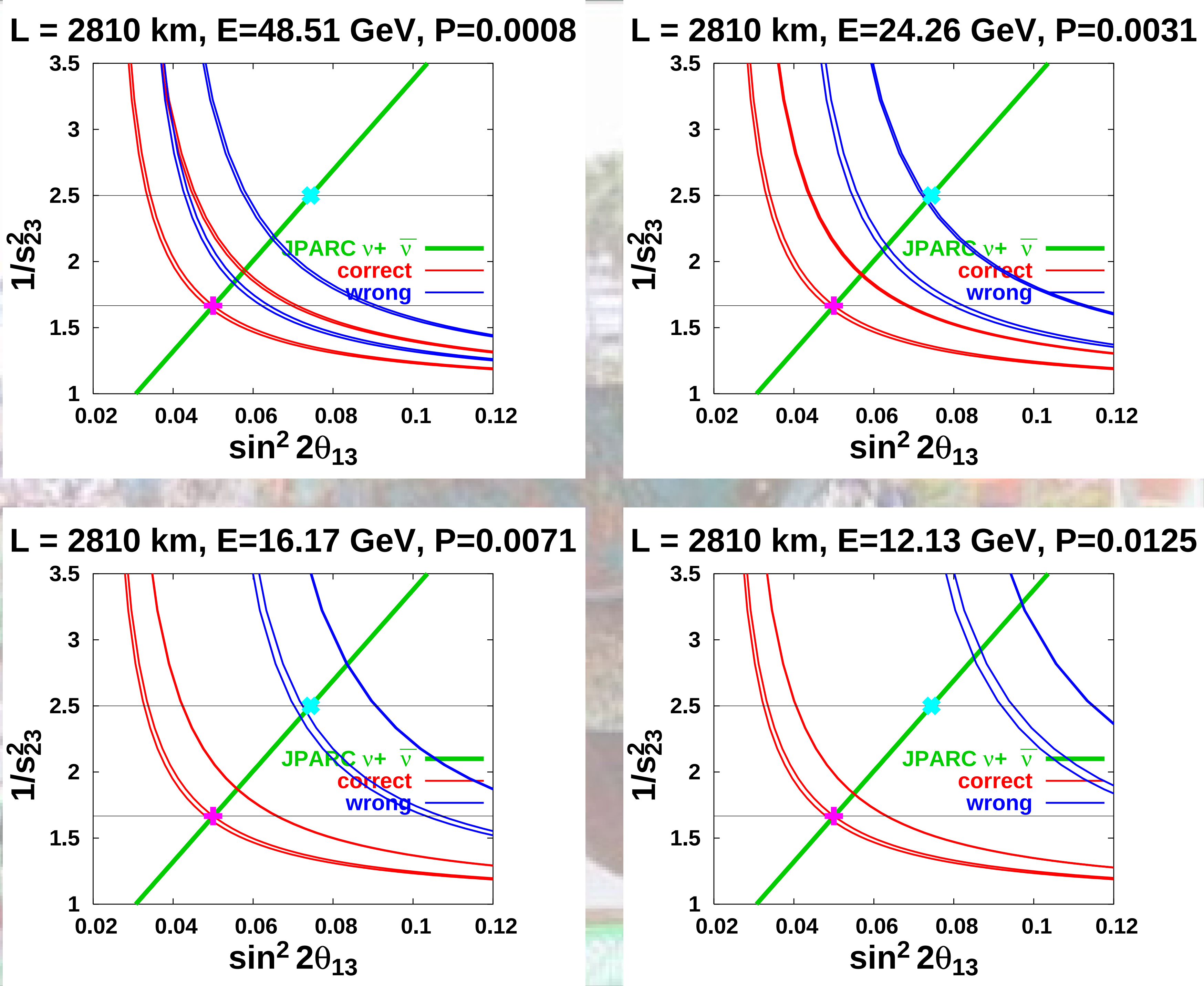

L = 3000 km, E=9.70 GeV, P=0.0282
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12


L = 3000 km, E=8.09 GeV, P=0.0388
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

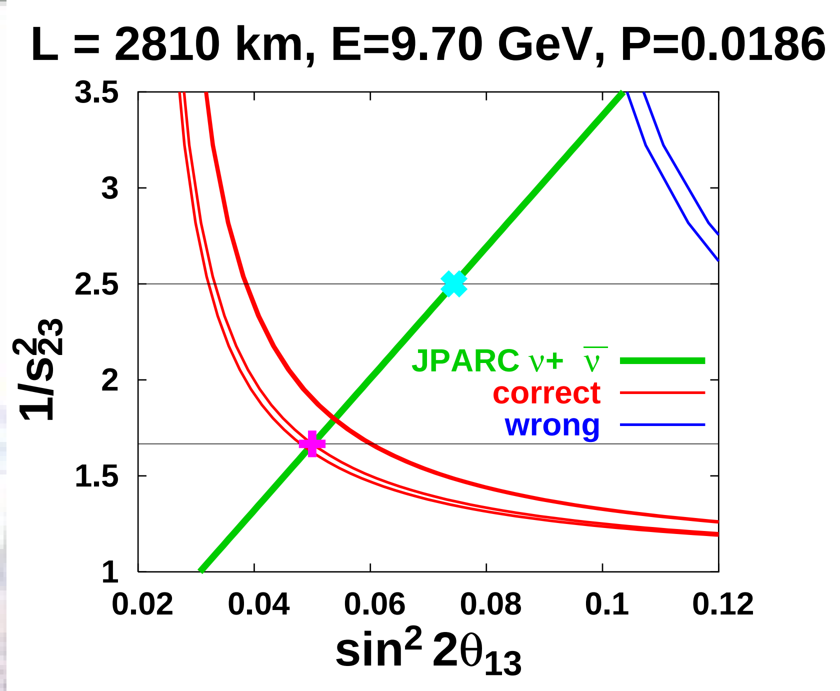

L = 3000 km, E=6.93 GeV, P=0.0491
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12

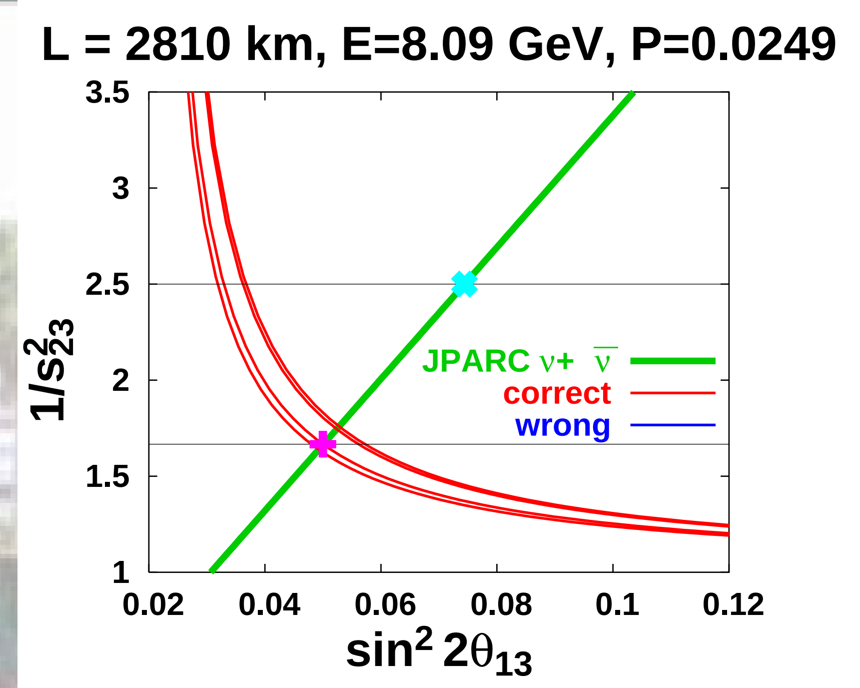


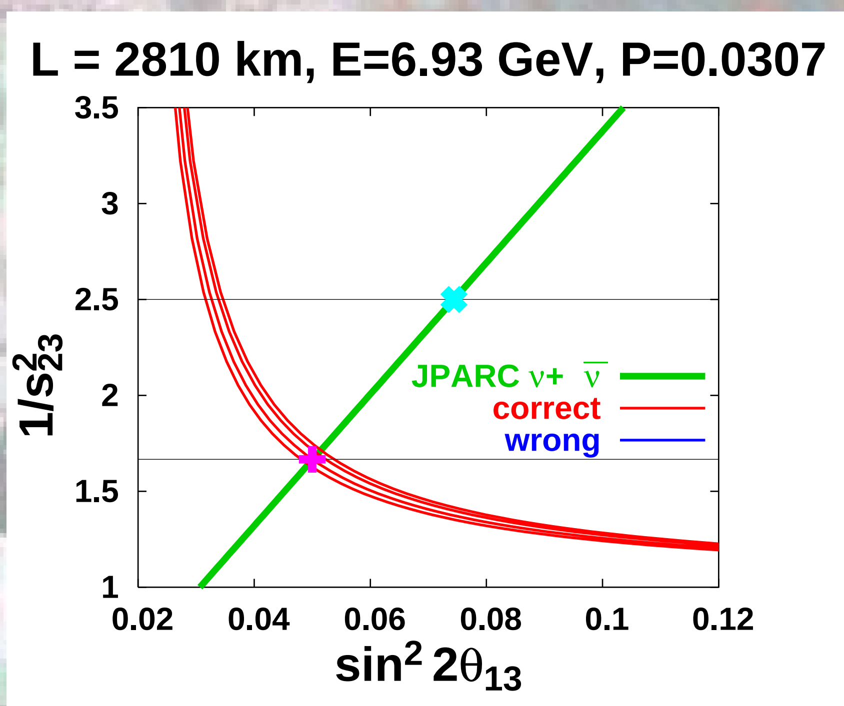
L = 732 km, E=11.80 GeV, P=0.0009
L = 732 km, E=5.90 GeV, P=0.0034
L = 732 km, E=3.93 GeV, P=0.0071
L = 732 km, E=2.95 GeV, P=0.0112
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12



L = 2810 km, E=48.51 GeV, P=0.0008
L = 2810 km, E=24.26 GeV, P=0.0031
L = 2810 km, E=16.17 GeV, P=0.0071
L = 2810 km, E=12.13 GeV, P=0.0125
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃
3.5
3
2.5
2
1.5
1
0.02
0.04
0.06
0.08
0.1
0.12


L = 2810 km, E=9.70 GeV, P=0.0186
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃


L = 2810 km, E=8.09 GeV, P=0.0249
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃


L = 2810 km, E=6.93 GeV, P=0.0307
JPARC ν+ ν̄
correct
wrong
1/s²₂₃
sin² 2θ₁₃