Degeneracy and strategies of LBL

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● Discussions on probabilities only w/o statistical and systematic errors

● Discussions in sect. 2 &3 assume JPARC $\nu + \bar{\nu}$ at @ Osc. Max.

→ scenarios 10 years from now

Based on hep-ph/0405005
hep-ph/0405222

Notation in this talk

Notations in this talk:

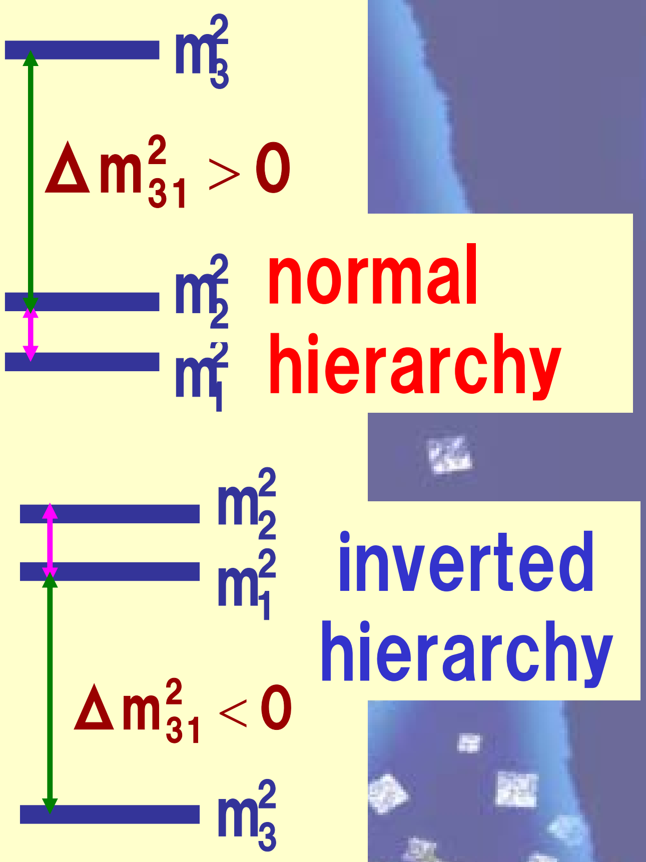
$$P \equiv P(\nu_\mu \to \nu_e)$$

$$\bar{P} \equiv P(\overline{\nu_\mu} \to \overline{\nu_e})$$

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E}$$

Oscillation Maximum (OM)

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E} = \frac{\pi}{2}$$



1. Introduction

Even if we know $P(\nu_\mu \rightarrow \nu_e)$ and $P(\overline{\nu_\mu} \rightarrow \overline{\nu_e})$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of θ_{13}, sign (Δm^2_{31}) and δ is difficult because of the 8-fold parameter degeneracy.

- intrinsic (δ, θ_{13}) degeneracy
- $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy
- $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

Plots in ($\sin^2 2\theta_{13}$, $1/s^2_{23}$) plane

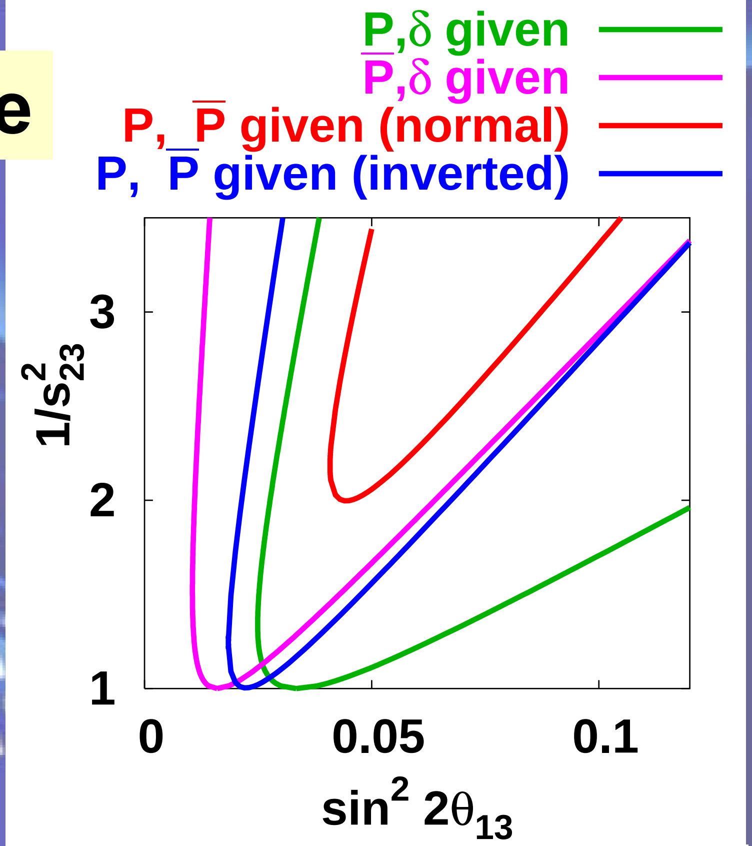
The way curves intersect is easy to see

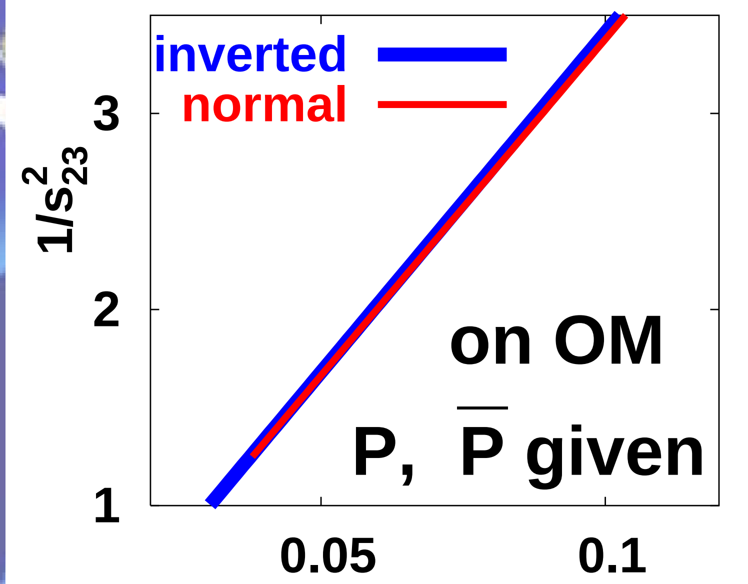
(P=const, δ=const)
($\bar{P}$=const, δ=const)
(P=const& $\bar{P}$=const' **off OM**)

hyperbolas (or ellipses)

(P=const& $\bar{P}$=const' **on OM**)

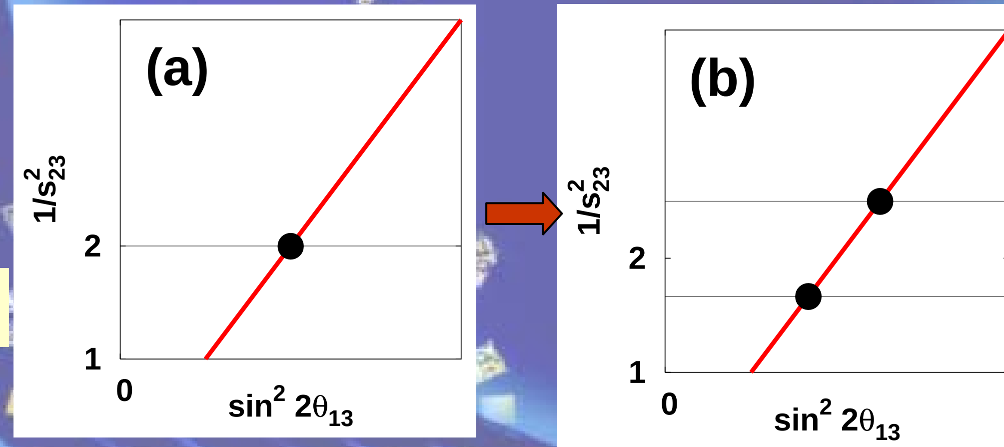
straight lines





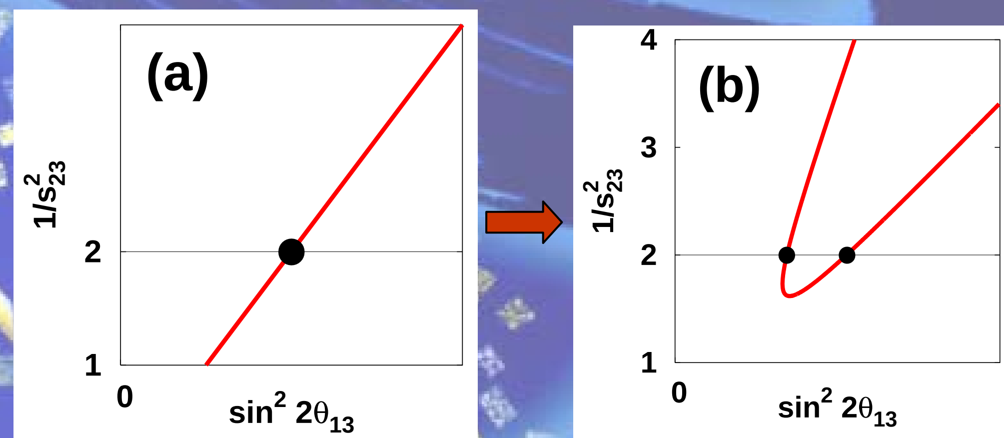
● $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy

$$(a)\cos 2\theta_{23} = 0 \rightarrow (b)\cos 2\theta_{23} \neq 0$$



● intrinsic (δ, θ_{13}) degeneracy

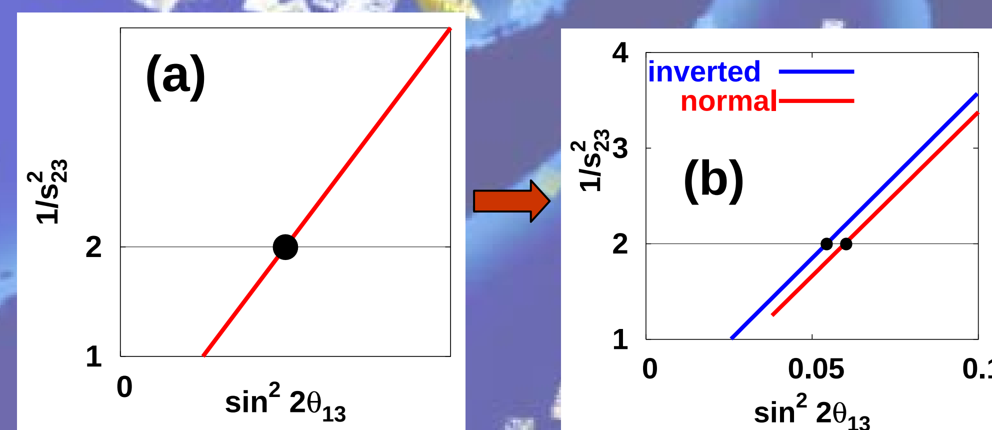
$$(a)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} = 0 \rightarrow (b)\frac{\Delta m^2_{21}}{|\Delta m^2_{31}|} \cong \frac{1}{35} \neq 0$$



● $\Delta m^2_{31} \Leftrightarrow -\Delta m^2_{31}$ degeneracy

$$(a)AL/2 = 0 \rightarrow (b)AL/2 \neq 0$$

$$A \equiv \sqrt{2}G_F N_e \cong 1/2000\text{km}$$



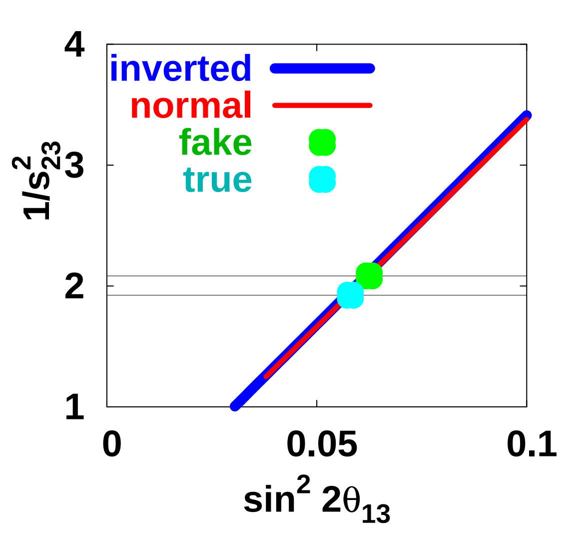
2. Determination of θ$_{13}$

Assumption: $\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured at JPARC (@OM, 4MW, HK) .

Question: Will that be enough to determine $|U_{e3}|$?

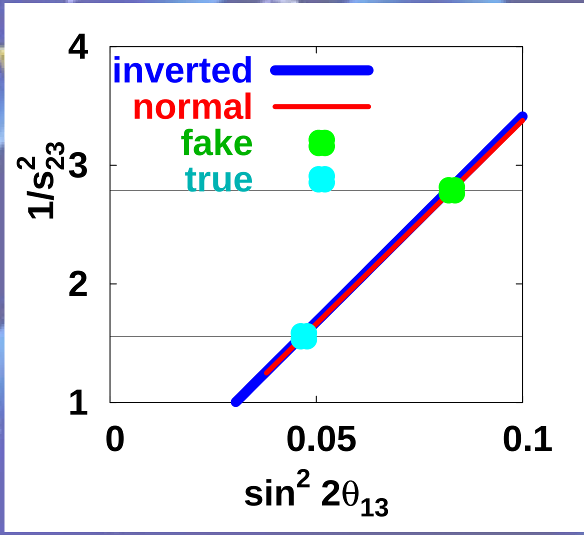
(1) $\sin^2 2\theta_{23} \simeq 1$ ➡ Yes!

JPARC $\nu + \overline{\nu}$ is almost enough, since (a) there is no intrinsic (δ, θ_{13}) degeneracy, and (b) sign(Δm^2_{31}) degeneracy is small.



(2) $\sin^2 2\theta_{23} < 1$ ➡ No!

Ambiguity due to $\theta_{23} \Leftrightarrow \pi/2 - \theta_{23}$ degeneracy is significant.



To resolve θ_{23} ambiguity, possible ways are:

(A)reactor measurement of θ_{13} $\quad \bar{\nu}_e \to \bar{\nu}_e$

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

(C) measurement of $\nu_e \to \nu_\tau$

The reference values used here are:

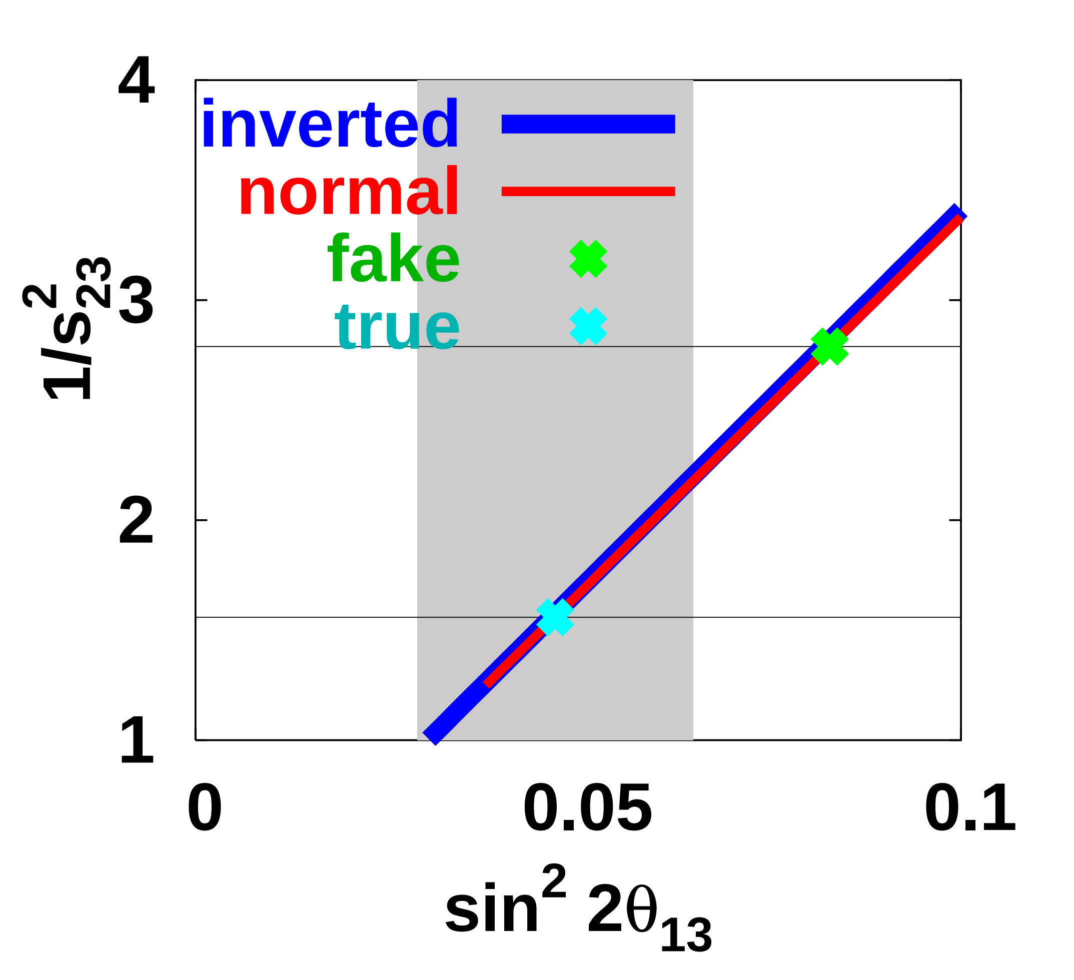
$\sin^2 2\theta_{23}=0.96$, $\sin^2 2\theta_{13}=0.05$, $\delta=\pi/4$, $\Delta m^2_{31}>0$

(A) reactor measurement of θ_{13} $\bar{\nu}_e \rightarrow \bar{\nu}_e$

One can resolve θ_{23} ambiguity at 90%CL.

↓

To compete with accelerator experiments, improvements in the sensitivity is necessary.



One possible way to improve sensitivity of reactor measurements (theorist's personal speculation)
O.Y. LENE3@Niigata, March 20, 2004

If one puts N near detectors and N far detectors with the same σ_u, then theoretically sensitivity becomes:

$$\min_{L_f, L_n} \left(\sin^2 2\theta_{13}\right)^{\text{sys only}}_{\text{limit}} = 2.8\,\sigma_u \quad @90\%\text{CL}$$

$$\chi^2 \Rightarrow N\chi^2$$

$$\min_{L_f, L_n} \left(\sin^2 2\theta_{13}\right)^{\text{sys only}}_{\text{limit}} = 2.8\sqrt{\frac{1}{N}}\,\sigma_u \quad @90\%\text{CL}$$

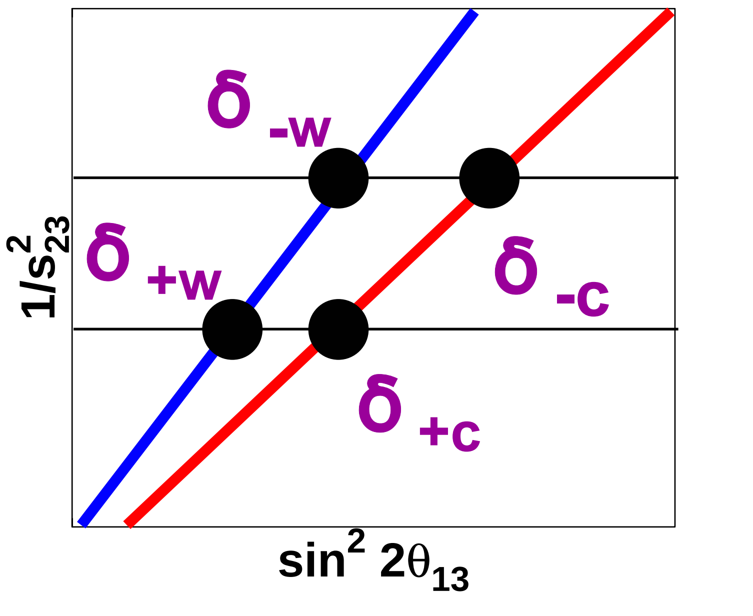
σ_u : the uncorrelated systematic error (~0.005 by optimistic estimate)

(B) LBL measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

Consider 3rd measurement of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$)

in addition to JPARC $\nu + \bar{\nu}$.

↓ (exaggerated figure)



correct assumption
wrong assumption
on mass hierarchy

The value of δ for each point can be deduced (up to $\delta \Leftrightarrow \pi - \delta$) from

$$\sin\delta = -\frac{P - f^2x^2 - g^2y^2}{2fgxy},$$

$$x \equiv s_{23}\sin 2\theta_{13},$$

$$y \equiv |\Delta m^2_{21}/\Delta m^2_{31}|\, c_{23}\sin 2\theta_{12},$$

$$f, \bar{f} \equiv \sin(\Delta \mp AL/2)/(1 \mp AL/2\Delta),$$

$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

Then from the equation for the probability of $\nu_\mu \to \nu_e$ (or $\nu_e \to \nu_\mu$) in the 3rd experiment

$$P_{true} = P\left(\sin^2 2\theta_{13}, \delta_{\pm[cw]}, s^2_{23}\right)$$

or

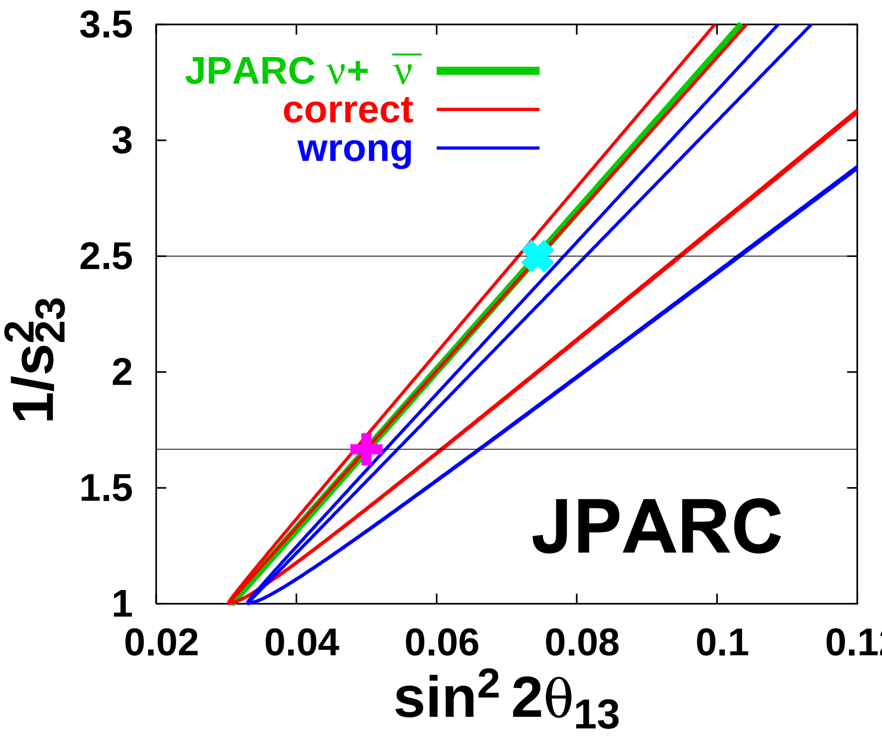
$$P_{true} = P\left(\sin^2 2\theta_{13}, \pi - \delta_{\pm[cw]}, s^2_{23}\right)$$

where

$$P_{true} \equiv P\left((\sin^2 2\theta_{13})_{true}, \delta_{true}, (s^2_{23})_{true}\right)$$

we can get a unique line (a hyperbola or an ellipse) in $(\sin^2 2\theta_{13}, 1/s^2_{23})$ plane for $\delta_{\pm[cw]}$ or $\pi - \delta_{\pm[cw]}$.

L = 295 km, E=1.19 GeV, P=0.0158



δ ⇔ π - δ ambiguity

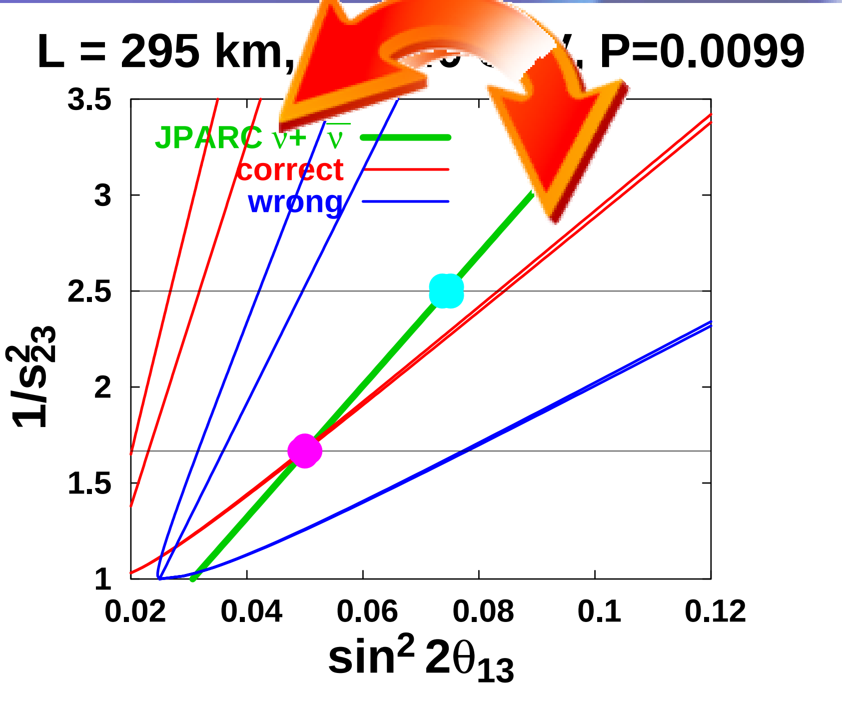
$\propto \cos\delta\,\cos\Delta$

Assuming for simplicity P>>C

$$C \equiv \left(\Delta m^2_{21}/\Delta m^2_{31}\right)^2 g^2 \sin^2 2\theta_{12}$$
$$g \equiv \sin(AL/2)/(AL/2\Delta)$$

$$\left.\frac{dX}{dY}\right|_{\delta} \cong -\frac{2\sqrt{PC}}{f^2}\cos(\delta+\Delta)$$

$$\left.\frac{dX}{dY}\right|_{\pi-\delta} \cong \frac{2\sqrt{PC}}{f^2}\cos(\delta-\Delta)$$



$$\left.\frac{dX}{dY}\right|_{\delta} - \left.\frac{dX}{dY}\right|_{\pi-\delta} \cong -\frac{4\sqrt{PC}}{f^2}\cos\delta\,\cos\Delta$$

Difference in the gradients is large for Δ=0 or π

● sign(Δm^2_{31}) ambiguity

(a) L: AL～L/1900km

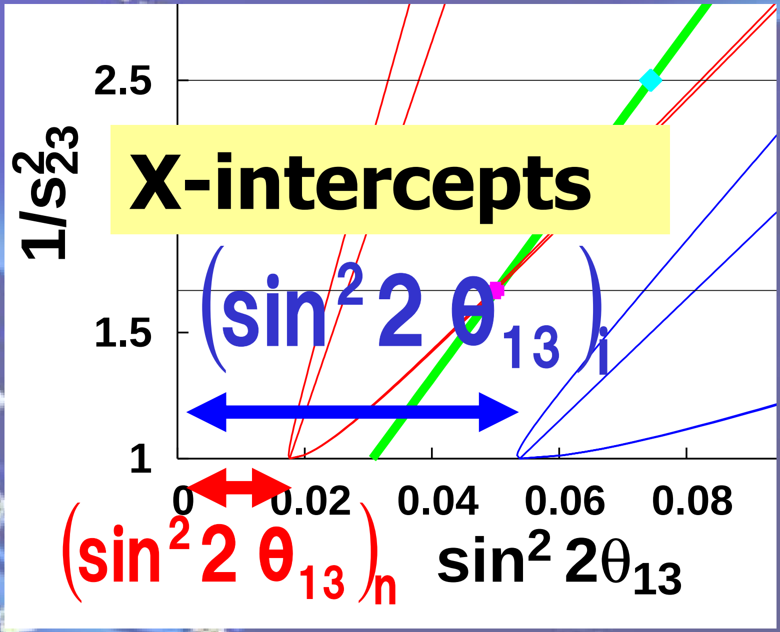
→ L>2000km is good to identify sgn(Δm^2_{31})

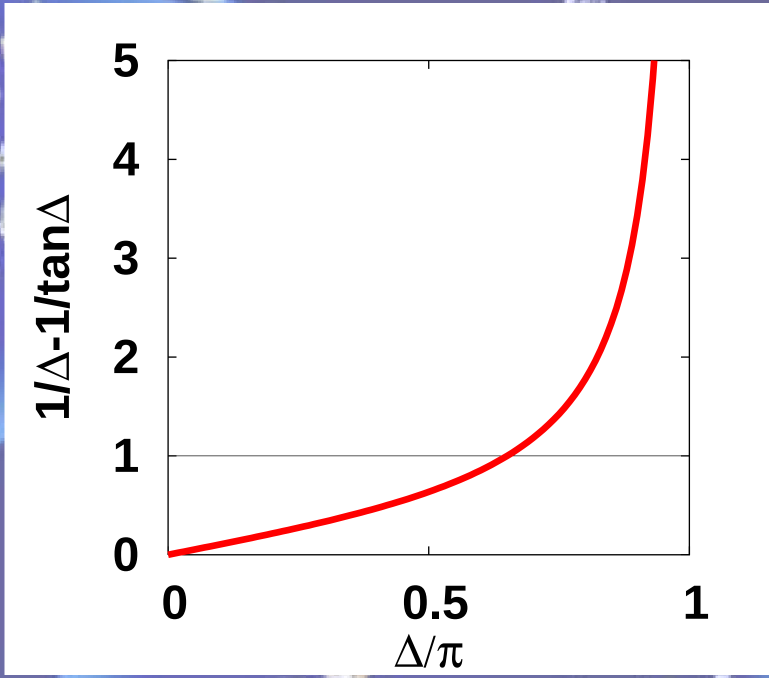
(b) E: → low energy is advantageous

$$\frac{\left(\sin^2 2\theta_{13}\right)_i}{\left(\sin^2 2\theta_{13}\right)_n} \simeq 1 + 2AL\left(\frac{1}{\Delta} - \frac{1}{\tan\Delta}\right)$$

Enhancement of matter effect for $\pi/2<\Delta<\pi$

In my personal opinion nova should run with lower E!





θ$_{23}$ ambiguity

Resolution of θ$_{23}$ ambiguity

(a) $f \equiv \frac{\sin(\Delta - AL/2)}{1 - AL/2\Delta}$ has to be small

(b) $|\cos(\delta + \Delta)|$ has to be large

↓

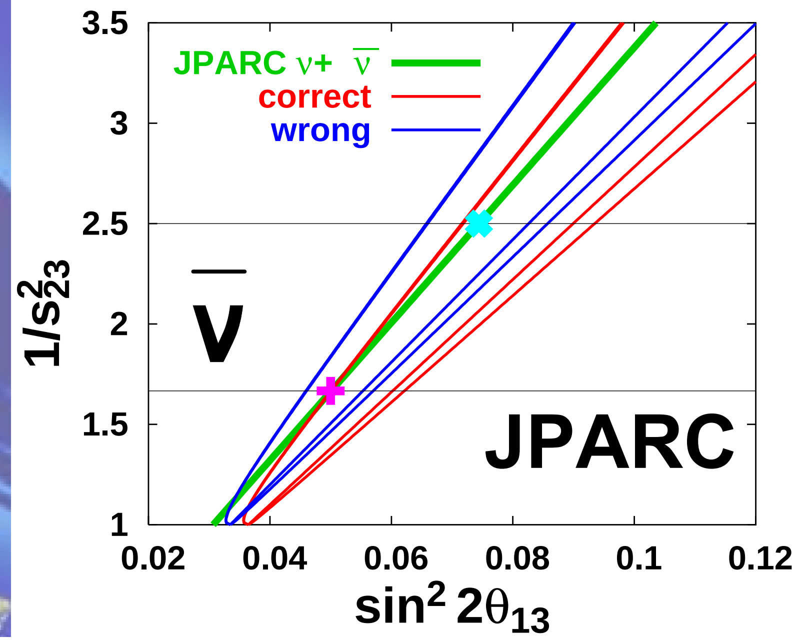
δ is unknown at first, so it is impossible to design to optimize this resolution.

It may happen that this ambiguity can be resolved as a byproduct.

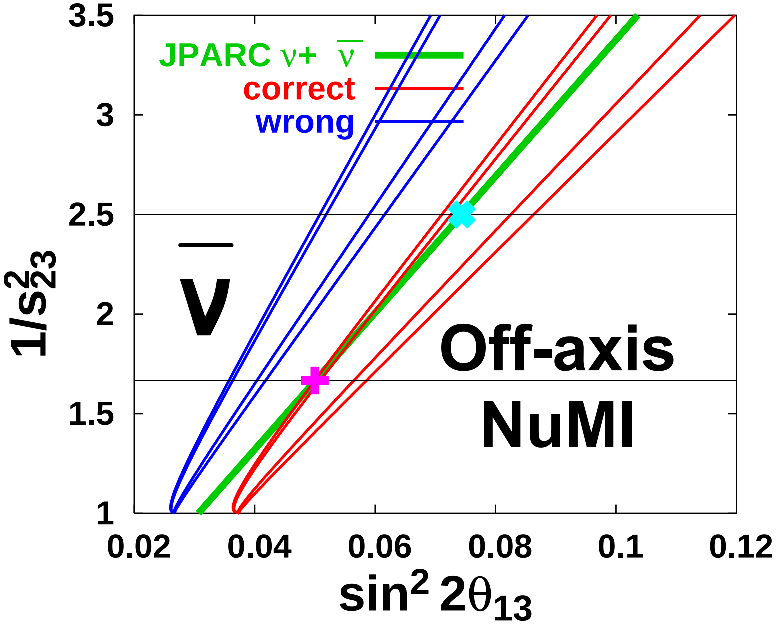
$2\frac{g}{f}xy|\cos(\delta + \Delta)|$

The situation doesn't change much for $\bar{\nu}_\mu \to \bar{\nu}_e$ if $\Delta \leqq \pi/2$.

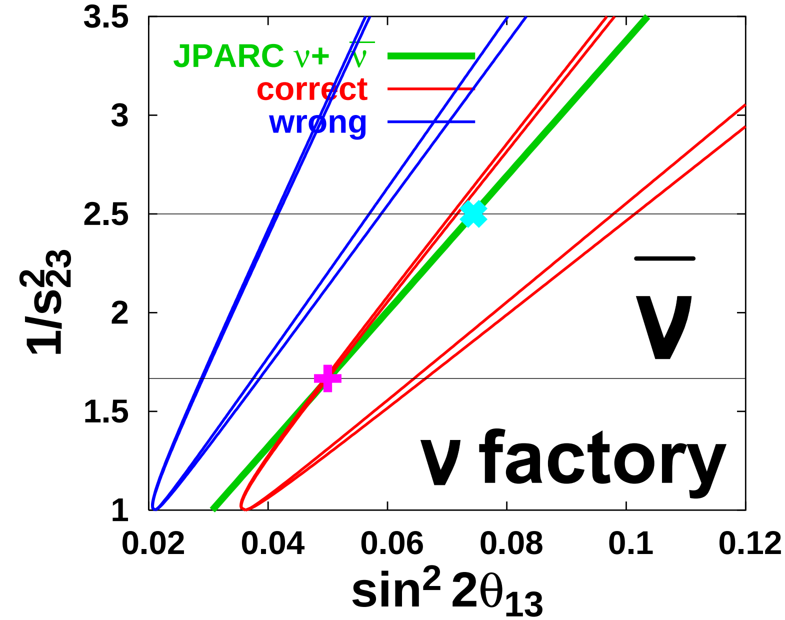
L = 295 km, E=1.19 GeV, P=0.0174



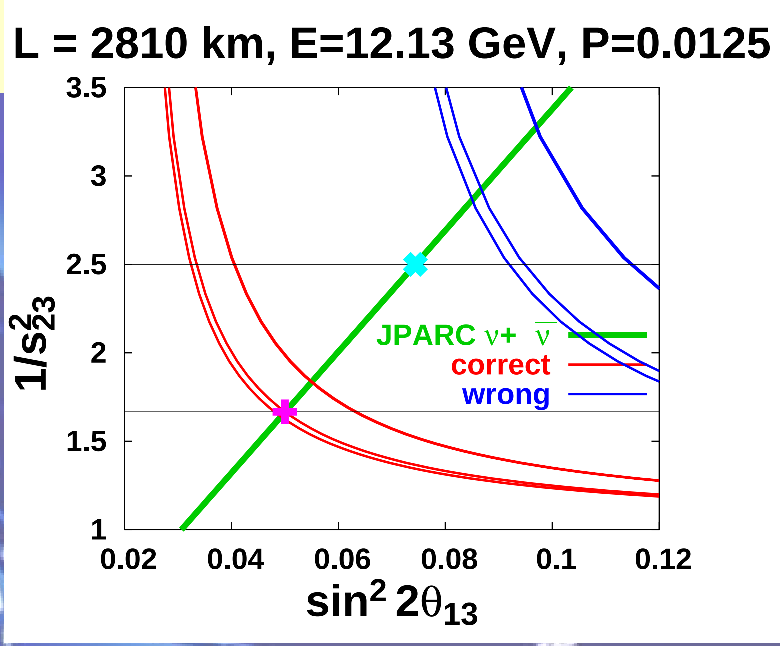
L = 730 km, E=1.97 GeV, P=0.0265



L = 3000 km, E=24.26 GeV, P=0.0029



(C) measurement of $\nu_e \to \nu_\tau$



Curves intersect with the JPARC line almost orthogonally.

- θ_{23} ambiguity may be resolved.
- $\delta \Leftrightarrow \pi - \delta$ ambiguity may be resolved.
- sign(Δm^2_{31}) ambiguity may be resolved .

This channel may be interesting to be combined with JPARC in the future.

3. δ

Assumption: at JPARC (@OM, 4MW, HK)

$\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ will be measured.

Question:

Will that be enough to determine arg(U_{e3})?

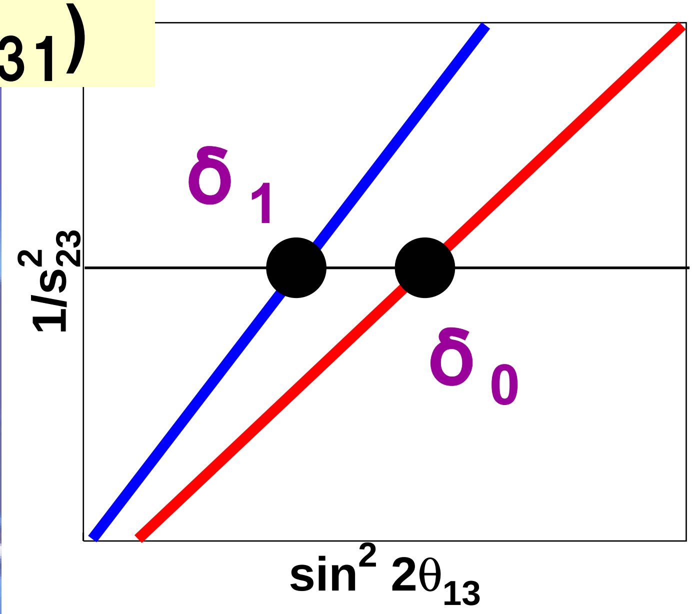
→ **Answer: In general no.**

Resolution of sign(Δm^2_{31}) ambiguity is important.

(1) Ambiguity due to sign(Δm^2_{31})

δ_0 : by correct assumption =true value

δ_1 : by wrong assumption on sign(Δm^2_{31})



Difference between δ_0 & δ_1 turns out to be large.

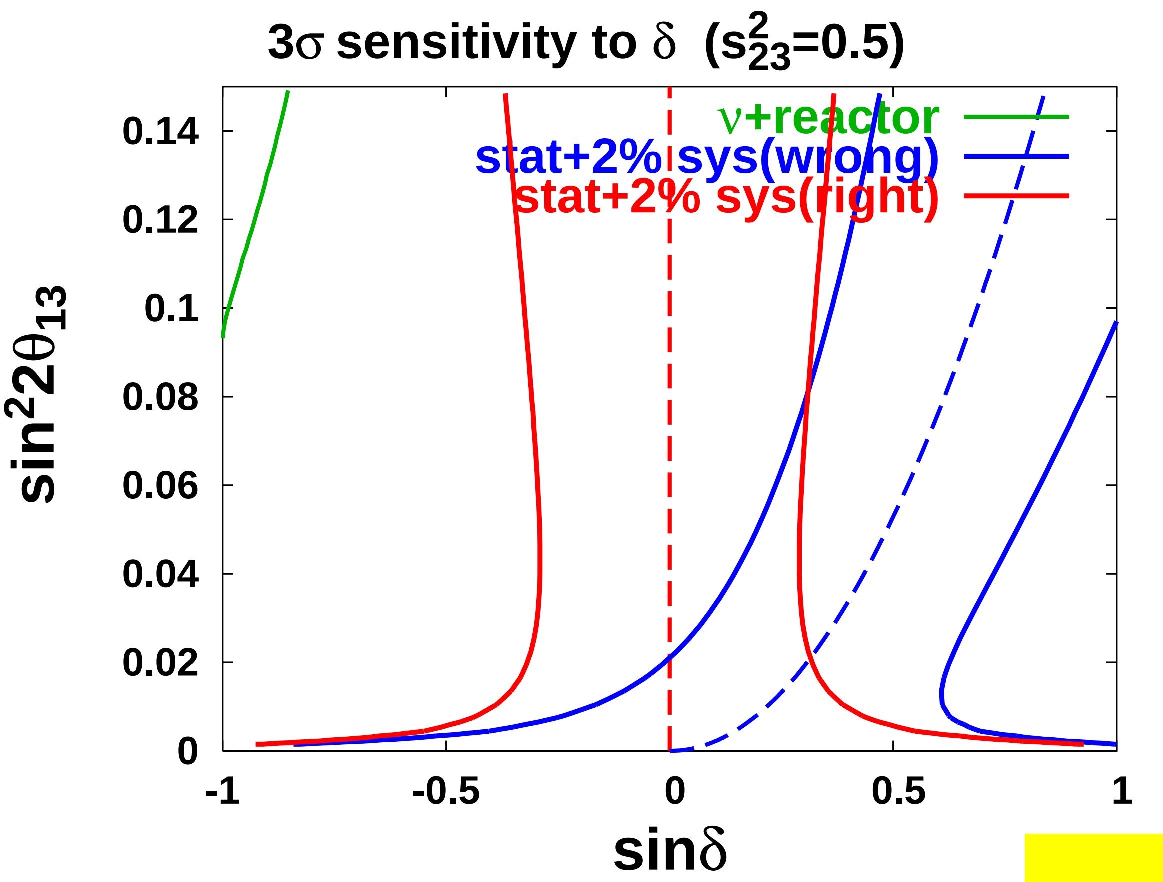
If δ_0=0, then $\sin\delta_1 \cong -2.2 \sin 2\theta_{13}$ at JPARC

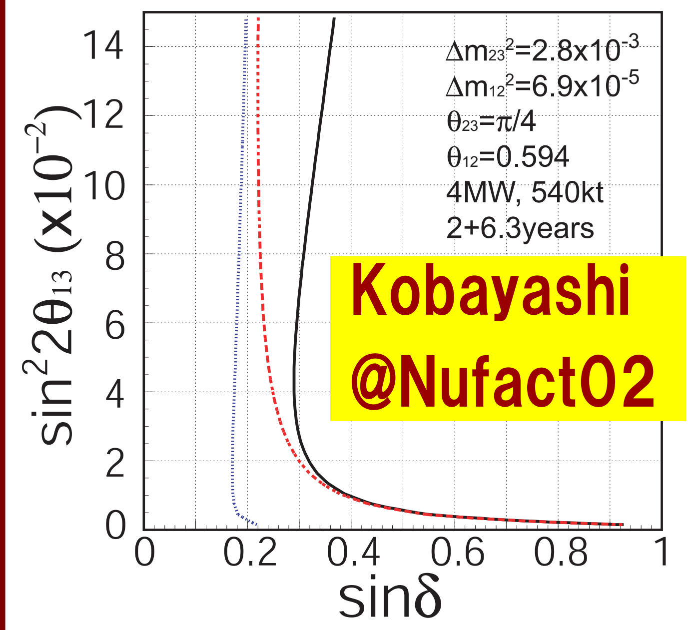
= -0.5 (if $\sin^2 2\theta_{13}$=0.05)

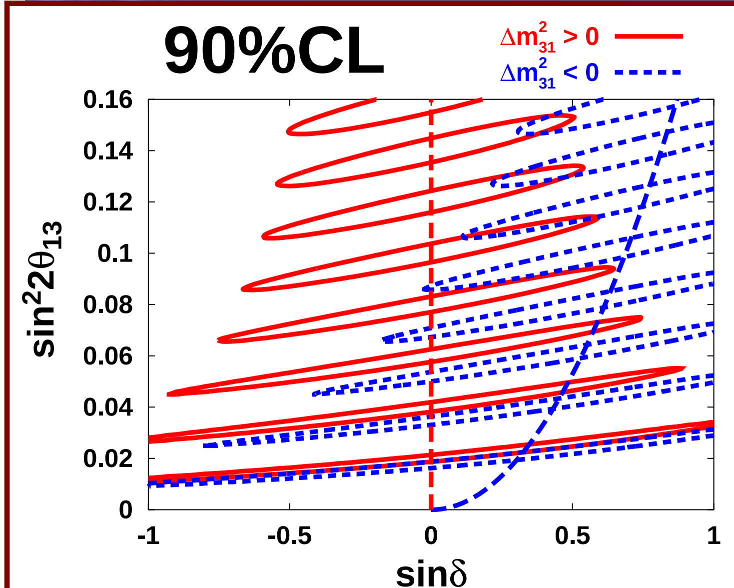
→ Identification of sign(Δm^2_{31}) is important.

3σ sensitivity to δ

Assuming $\Delta m^2_{31}>0$

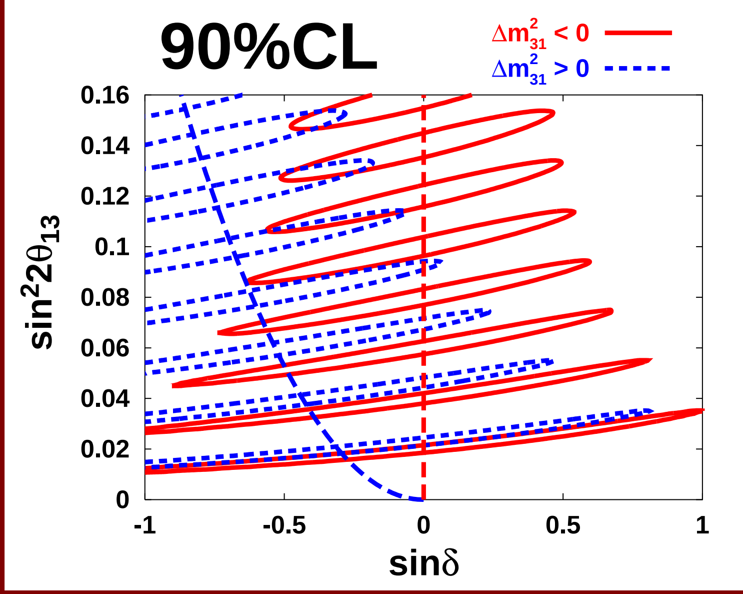


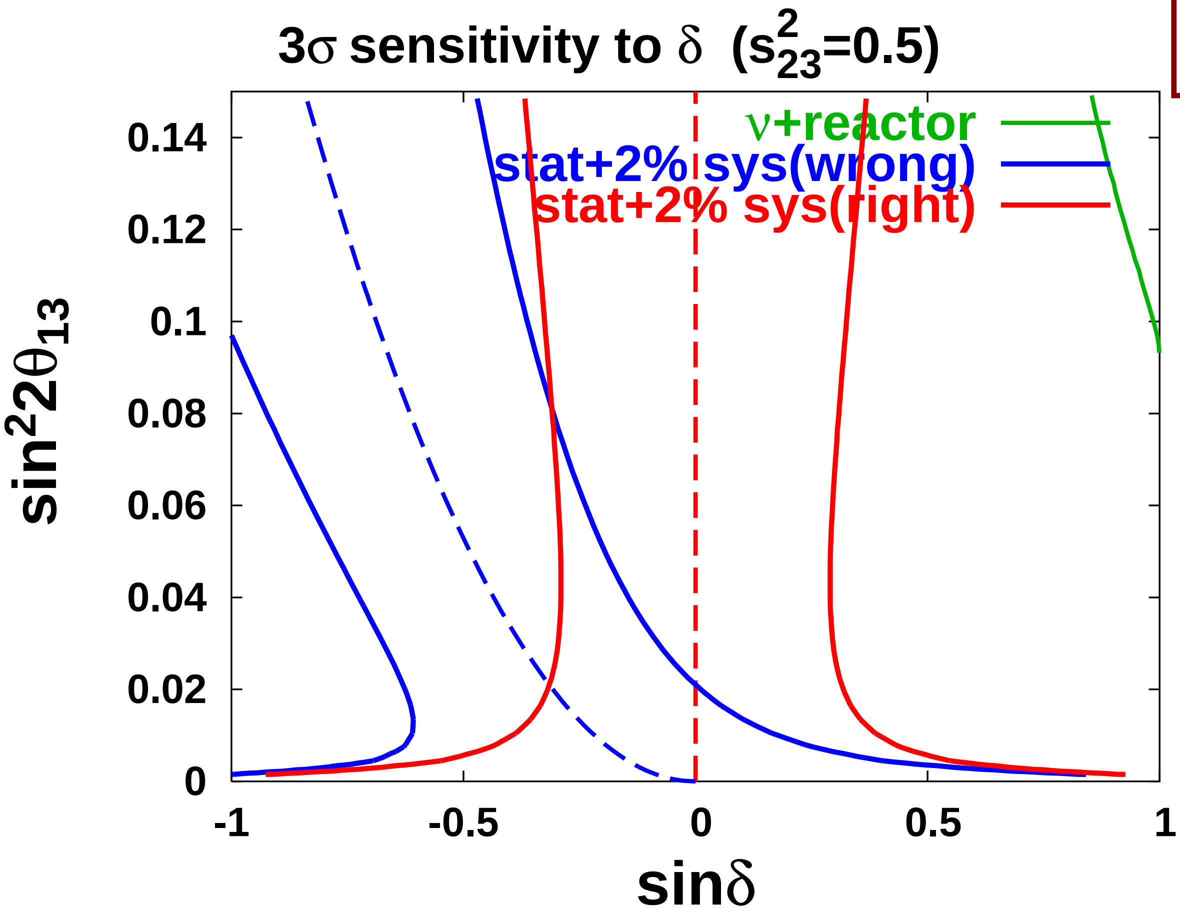




↑modified from Minakata-Sugiyama (PLB580,216)

Assuming $\Delta m^2_{31}<0$





↑modified from Minakata-Sugiyama (PLB580,216)

(2) Ambiguity of δ due to θ_{23} :

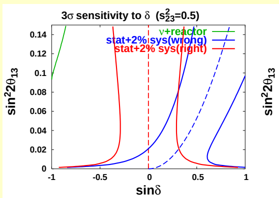
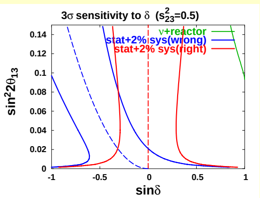
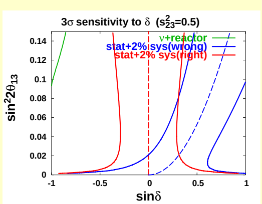
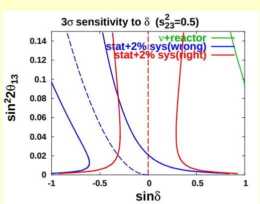
$$|\sin\delta_2| \cong \frac{1}{200}\frac{|\cot 2\theta_{23}|}{t_{23}}\frac{1}{\sqrt{\sin^2 2\theta_{13}}} \leq \frac{1}{500}\frac{1}{\sqrt{\sin^2 2\theta_{13}}}$$

$$0.9 \leq \sin^2 2\theta_{23} \leq 1.0 \; (\nu_{atm})$$

→ The Ambiguity due to θ_{23} is not serious.

4.Summary

Phase II → δ ; Phase I → θ_{13}

δ \ θ_{13}	$\sin^2 2\theta_{23} \simeq 1$	$\sin^2 2\theta_{23} < 1$
$(\sin\delta, \sin^2 2\theta_{13})$ outside of red or blue lines  	JPARC@OM $\nu_\mu \to \nu_e$ & $\bar{\nu}_\mu \to \bar{\nu}_e$ is almost enough	In addition to JPARC ν & $\bar{\nu}$@OM, $\bar{\nu}_e \to \bar{\nu}_e$ or $\nu_e \to \nu_\tau$ is necessary to resolve θ_{23} ambiguity
$(\sin\delta, \sin^2 2\theta_{13})$ inside of red or blue lines  	In addition to JPARC ν & $\bar{\nu}$@OM, LBL w/ L>~1000km is necessary to resolve sign(Δm^2_{23}) ambiguity	In addition to JPARC ν & $\bar{\nu}$@OM, (A) $\bar{\nu}_e \to \bar{\nu}_e$ or $\nu_e \to \nu_\tau$ is necessary to resolve θ_{23} ambiguity (B) LBL w/ L>~1000km is necessary to resolve sign(Δm^2_{23}) ambiguity

It is important

- for determination of θ_{13}

  to resolve θ_{23} ambiguity

  if $\sin^2 2\theta_{23} < 1$.

- for determination of δ

  to resolve sign (Δm^2_{31}) ambiguity.

If nova runs with lower E, then it will become complementary to JPARC, and only in this case it will play an important role.