Phenomenology of $\theta_{13}$

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JPS meeting
11 September 2012@Kyoto Sangyo Univ.
1. Introduction

Framework of 3 flavor $\nu$ oscillation

Mixing matrix

Functions of mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and CP phase $\delta$

3 mixing angles have been measured:

$\nu_{\text{solar}} + $KamLAND (reactor)

$\nu_{\text{atm}} + $K2K,MINOS (accelerators)

$\nu_{\text{CHOOZ}} + $Daya Bay+Reno (reactors), T2K+MINOS, others

Both hierarchy patterns are allowed

Normal hierarchy

Inverted hierarchy

$3 \times 3$ mixing angles have been measured:

$\theta_{12} \approx \frac{\pi}{6}, \Delta m_{21}^2 \approx 8 \times 10^{-5} \text{eV}^2$

$\theta_{23} \approx \frac{\pi}{4}, |\Delta m_{32}^2| \approx 2.5 \times 10^{-3} \text{eV}^2$

$\theta_{13} \approx \frac{\pi}{20}$
One hint at nu2012: $\theta_{23}$ appears to be nonmaximal

Forero, Tortola, Valle arXiv:1205.4018 (nu2012 data included)
\pi/4 - \theta_{23} < 0 \text{ is preferred}

Forero, Tortola, Valle arXiv:1205.4018
Octant of $\theta_{23}$ ($\pi/4 - \theta_{23} > 0$?) appears to be subtle

$\pi/4 - \theta_{23} > 0$ is preferred

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno
Info on MH seems necessary

$\Delta m^2$ and $\sin^2 \theta_{23}$ with reactor constraint

- Normal hierarchy (NH)
  - $\sin^2 \theta_{23}$: $0.391 - 0.619$ (90% C.L.)
  - $\Delta m^2_{32}$: $2.66 \pm 0.15 \times 10^{-3} eV^2$ (1\sigma)

-Inverse hierarchy (IH)
  - $\sin^2 \theta_{23}$: $0.393 - 0.630$ (90% C.L.)
  - $\Delta m^2_{32}$: $2.66 \pm 0.17 \times 10^{-3} eV^2$ (1\sigma)

- $\chi^2$ plots:
  - Free $\theta_{13}$
  - Fixed reactor $\theta_{13}$
  - 99% C.L.
  - 90% C.L.
  - 68% C.L.
A word on theory: Simple theoretical ansatz to predict $\theta_{13}$ successfully

◆ Anarchy
Hall, Murayama, Weiner, PRL 84 (2000) 2572

$\sin^2 2\theta_{13} \sim 0.1$ $\sin^2 2\theta_{23} \sim 1$

◆ Quark-lepton complementarity
Minakata, Smirnov, PR D70 (2004) 073009

$\theta_{12} + \theta_C = 45$ deg

$\theta_{13} = 8.9$ deg
$\theta_{12} = 35.4$ deg
$\theta_{23} = 42.1$ deg

Dighe, Goswami, Roy PR D76 (2007) 096005
2. Parameter degeneracy

\[ U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & \varepsilon \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \]

Both mass hierarchies are allowed

Next task is to measure\[ \text{sign}(\Delta m^2_{31}), \pi/4-\theta_{23} \text{ and } \delta. \]

To determine \( \delta \), accelerator long baseline experiments with \( \nu_{\mu} \rightarrow \nu_{e} \) and \( \bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e} \) are necessary.
Parameter degeneracy

Even if we know $P(v_\mu \rightarrow v_e)$ and $P(\overline{v}_\mu \rightarrow \overline{v}_e)$ in a long baseline accelerator experiments with approximately monoenergetic neutrino beam, precise determination of $\theta_{13}$, $\theta_{23}$, $\text{sign}(\Delta m^2_{31})$, and $\delta$ is difficult because of the 8-fold parameter degeneracy.
In this plot, the region of $P=\text{const}$ or $\bar{P}=\text{const}$ is described by quadratic curves (hyperbolic or elliptic).
P=const: region bounded by a hyperbola

P=const': region bounded by a hyperbola

Oscillation Maximum:

$$\Delta \equiv \frac{\Delta m_{31}^2}{4E} = \frac{\pi}{2}$$
○ octant degeneracy
\[ \theta_{23} \leftrightarrow \pi/2 - \theta_{23} \]
\[(a) \cos 2\theta_{23} = 0 \rightarrow (b) \cos 2\theta_{23} \neq 0\]

○ intrinsic degeneracy
\[(\delta, \theta_{13})\]
\[(a) \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} = 0 \rightarrow (b) \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} \approx \frac{1}{35} \neq 0\]

○ sign degeneracy
\[\Delta m_{31}^2 \leftrightarrow -\Delta m_{31}^2\]
\[(a) AL/2 = 0 \rightarrow (b) AL/2 \neq 0\]
\[A \equiv \sqrt{2G_FN_e} \approx 1/2000\text{km}\]
In total we have 8-fold parameter degeneracy.

Each point has different value of $\delta$.

For precise measurements of $\delta$, one has to resolve parameter degeneracy.
Differences in values of CP phases

\[ \theta_{13} := \theta_{13}(\text{true}), \quad \theta_{13}' := \theta_{13}(\text{false}) \]
\[ \delta := \delta(\text{true}), \quad \delta' := \delta(\text{false}) \]

sign degeneracy

\[
\sin^2 2\theta_{13}' = \sin^2 2\theta_{13} \tan^2 \theta_{23} + \frac{\alpha^2 g^2 \sin^2 2\theta_{12}}{f \bar{f}} (1 - \tan^2 \theta_{23}),
\]
\[
\sin 2\theta_{13}' \sin \delta' = \sin 2\theta_{13} \sin \delta + \frac{\alpha g (f - \bar{f}) \sin 2\theta_{12}}{f \bar{f}} \cot 2\theta_{23} \frac{\sin \Delta}{\sin \Delta},
\]

octant degeneracy

\[
x'^2 = \frac{x^2 (f^2 + \bar{f}^2 - f \bar{f}) - 2y_g (f - \bar{f}) x \sin \delta \sin \Delta}{f \bar{f}},
\]
\[
x' \sin \delta' = x \sin \delta \frac{f^2 + \bar{f}^2 - f \bar{f}}{f \bar{f}} - \frac{x^2}{\sin \Delta} \frac{f^2 + \bar{f}^2}{f \bar{f}} \frac{f - \bar{f}}{2y_g}.
\]

Sign degeneracy is more serious than octant one, because \( \sin\delta(\text{sign}) = 0 \Rightarrow \sin\delta'(\text{sign}) = O(1) \neq 0 \)

**NB:** At T2K

\[ |\Delta m^2_{31}| \frac{L}{4E} = \frac{\pi}{2} \Rightarrow \sin\delta(\text{intrinsic}) = \sin\delta'(\text{intrinsic}) \]

Resolution of sign degeneracy is important for CP measurement

\( \sin^2 2\theta_{13} = 0.1 \) at T2K

\[ \sin\delta, \sin\delta' \]

NH assumed
To solve parameter degeneracy, various combinations have been proposed:

(A) LBL measurement at \( |\Delta m_{31}^2| L/4E = \pi /2 \)
\[ \rightarrow \text{hyperbola shrinks to a straight line} \]
(B) reactor measurement of \( \theta_{13} \) \( \bar{v}_e \rightarrow \bar{v}_e \)
\[ \rightarrow \text{depends only on } \theta_{13} \]
(C) LBL measurement of \( v_\mu \rightarrow v_e \) (or \( v_e \rightarrow v_\mu \))
\[ \text{with different } L/E \]
(D) measurement of \( v_e \rightarrow v_\tau \)
Current status of appearance experiments

\[ \Delta m^2 > 0 \]
- MINOS Best Fit
- 68% C.L.
- 90% C.L.
- CHOOZ 90% C.L.

\[ 2\sin^2\theta_{23} = 1 \text{ for CHOOZ} \]

\[ \Delta m^2 < 0 \]
- MINOS
- 8.2 \times 10^{20} \text{ POT}

\[ \sin^2(2\theta_{13}) \]

\[ \delta_{CP} \]

arXiv:1108.0015

Sakashita@ICHEP2012
Current status: T2K+atm+reactors

90%CL (NH)

90%CL (IH)

atm 90%CL

Best fit (NH)

Best fit (IH)

reactors 90%CL

Allowed region from $P(\nu_\mu \rightarrow \nu_e)$ of T2K at best-fit & 90%CL (w/ Sakashita@ICHEP2012)

Error is large→needs more statistics & $P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e)$ to improve
Allowed region from $P(v_\mu \rightarrow v_\tau)$ of MINOS at best-fit & 90%CL (w/ arXiv:1108.0015 data)
3. Future LBL experiments

To perform precise measurements of $\theta_{13}$ and $\delta$, one has to have a lot of numbers of events to improve statistical errors.

→ We need high intensity beam

Candidates for high intensity beam in the future:

- (conventional) superbeam
  \[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
  \[ \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \]

- neutrino factory
  \[ \mu^+ \rightarrow e^+ + \nu_e + \nu_\mu \]
  \[ \mu^- \rightarrow e^- + \bar{\nu}_e + \bar{\nu}_\mu \]

- beta beam
  \[ \frac{6}{2} \text{He} \rightarrow \frac{6}{3} \text{Li} + e^- + \bar{\nu}_e \]
  \[ \frac{18}{10} \text{Ne} \rightarrow \frac{18}{9} \text{F} + e^+ + \nu_e \]
<table>
<thead>
<tr>
<th>Project</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>superbeam</strong></td>
<td></td>
</tr>
<tr>
<td>T2K phase II</td>
<td>(2.2MW+HK(+Okinoshima), E~1GeV, L=295km, 658km)</td>
</tr>
<tr>
<td>NOvA (FNAL ➔ Ash River (MN))</td>
<td>E~2GeV, L=810km</td>
</tr>
<tr>
<td>LBNE (FNAL ➔ Homestake)</td>
<td>E~a few GeV, L=1290km</td>
</tr>
<tr>
<td>CN2PY (CERN ➔ Pyhasalmi)</td>
<td>E~several GeV, L=2300km</td>
</tr>
<tr>
<td><strong>neutrino factory</strong></td>
<td>(E(_\nu)<del>20GeV, L</del>4000km)</td>
</tr>
<tr>
<td><strong>beta beam</strong></td>
<td>(E(_\nu)=0.5-1.5GeV, L~130km)</td>
</tr>
</tbody>
</table>

*Future LBL exp. (under construction / proposed)*
<table>
<thead>
<tr>
<th>Project</th>
<th>Type</th>
<th>Size/Location</th>
<th>Separation of IH and NH</th>
<th>Pre-requisite and date of achievement</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DayaBay II</td>
<td>reactor 60km</td>
<td>20 kt LS</td>
<td>3 σ in 8 years</td>
<td>R&amp;D on E-resolution 2020</td>
<td>Karsten Heeger at Neutrino 2012</td>
</tr>
<tr>
<td>ICAL@INO</td>
<td>atmospheric</td>
<td>50 kt MID (RPCs)</td>
<td>2.7 σ in 10 years</td>
<td>2027</td>
<td>Sandhya Choubey at Neutrino 2012</td>
</tr>
<tr>
<td>HyperK</td>
<td>atmospheric</td>
<td>1 Mt Water Cerenkov</td>
<td>3 σ in 5 years</td>
<td>2027/28</td>
<td>HyperK LOI Sandhya Choubey at Neutrino 2012</td>
</tr>
<tr>
<td>T2HK</td>
<td>LBL accel. 295 km</td>
<td>1 Mt Water Cerenkov</td>
<td>0.3 σ in 10 years</td>
<td>2028</td>
<td>Masashi Yokoyama at Neutrino 2012</td>
</tr>
<tr>
<td>PINGU</td>
<td>atmospheric</td>
<td>Ice (South pole)</td>
<td>3...11 σ in 5 years</td>
<td>feasibility study ongoing, understanding of resolution and systematics on atmospherics Around 2020 if it works.</td>
<td>Uli Katz at neutrino Town meeting</td>
</tr>
<tr>
<td>GLADE</td>
<td>LBL accel. 810 km</td>
<td>LAr 5 kt</td>
<td>In combination with NOvA and T2K: ≤ 2 σ</td>
<td>Letter-of-Intent</td>
<td>Jenny Thomas at neutrino Town meeting</td>
</tr>
<tr>
<td>NOvA</td>
<td>LBL AshRiver 810 km</td>
<td>TASSD 14 kt</td>
<td>0...3 σ in 6 years depending on δ</td>
<td>Full operation in 2014 2020</td>
<td>Ryan Patterson at Neutrino 2012</td>
</tr>
<tr>
<td>LBNE</td>
<td>LBL Homestake LBL Soudan LBL AshRiver</td>
<td>LAr 10 kt LAr 15 kt LAr 30 kt</td>
<td>1.5...7 σ in 10 y 0...3 σ in 10 y 0.5...5 σ in 10 y</td>
<td>2030</td>
<td>Bob Swoboda at Neutrino 2012</td>
</tr>
<tr>
<td>LBNO</td>
<td>LBL accel. 2300 km</td>
<td>LAr 20 kt</td>
<td>&gt;5σ in a few y.</td>
<td>2023 + If decision in 2015</td>
<td>André Rubbia at Neutrino 2012</td>
</tr>
<tr>
<td>LENA</td>
<td>LBL accel. 2300 km</td>
<td>Liq. Scint. 50 kt</td>
<td>5 σ in 10 years</td>
<td>2028 + number of years to the decision</td>
<td>Lothar Oberauer at Neutrino 2012</td>
</tr>
<tr>
<td>Neutrino Factory</td>
<td>LBL accel. 2000 km</td>
<td>MIND 100kton</td>
<td>&gt;&gt; 5 σ</td>
<td></td>
<td>Ken Long at Neutrino 2012</td>
</tr>
</tbody>
</table>
Future exp. vs MH

NB: JPARC+HK+Okinoshima is missing due to uncertainty in schedule

Year

Significance of MH

2010 2020 2030 2040

T2HK
HK-atm
LBE
LENA
INO-atm
DB-II
Nova
PINGU
LBNO

Now
T2K&Nova

Huber et al., arXiv:0907.1896v1

90%CL
Atmospheric $\nu@PINGU$

Doug Cowen, NuSky, ICTP, June 2011

IceCube $\rightarrow$ DeepCore $\rightarrow$ PINGU

- ~20 additional strings within DeepCore
- lower threshold to few GeV
- ~10 Mt effective volume
- construction within 1 yr, ~$25\ M$

Minakata@v2012

Akhmedov-Razzaque-Smirnov June 12

MH resolution 3$\sigma$-11$\sigma$ in 5 years!

$\sigma_E=2$ GeV
$\sigma_\theta=11.25^o$

$\nu^H - N^\nu_H/\langle N^\nu_H \rangle^{1/2}$ [PINGU 1 yr]

$\nu^H - N^\nu_H/\langle N^\nu_H \rangle^{1/2}$ [PINGU 1 yr] Smoothed
Mass Hierarchy and Reactor $\bar{\nu}_e$ Oscillation

Daya Bay II

Site Investigation

candidate site (~60km)

Haiteng

Daya Bay

Mass Hierarchy Sensitivity

50k events = 20 kton, 3 years → 96%
100k events → 3σ

Ref: Y. Wang, J. Cao, et al nuTurn 2012

Sub-1% precision 3-ν oscillation physics in $\Delta m^2_{12}$, $\Delta m^2_{23}$, and $\sin^2 \theta_{12}$ possible

Karsten Heeger, Univ. of Wisconsin

Neutrino2012, Kyoto, June 4, 2012
Future LBL plans using J-PARC

Current: T2K
J-PARC ~0.75MW
+ 50kt WC @ 295km 2.5°

LoI: The Hyper-Kamiokande Experiment
arXiv:1109.3262v1

J-PARC+LAr @ Okinoshima
L=658km OA=0.78deg

J-PARC P32 (LAr TPC R&D), arXiv:0804.2111
Mass Hierarchy

Hyper-Kamiokande LOI, arXiv:1109.3262v1 [hep-ex]
Comparison of Phase 1 Sensitivities to Mass Hierarchy and CP Violation

Preliminary: LBNE Physics Working Group

5 years neutrino + 5 years antineutrino
European sites: LAGUNA-LBNO

Three far sites considered in details

- Large Water Cerenkov Detector. CERN-Fréjus is a short baseline. It offers good synergy for enhanced physics reach with β-beam at γ=100

- Liquid Argon TPC & magnetized iron + Liquid Scintillator detectors. CERN-Pyhäsalmi is the longest baseline. It offers good synergy for enhanced physics reach with a NF

- [CNGS is an existing beam but is considered at lower priority (missing near detector, limited power upgrade scenarios)]
CP2PY

Rubbia@ν2012

MH determination

```
Δχ²

2.25e+20 pots
```

"conclusive" even with modest exposure

PRELIMINARY

CPV discovery

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Δχ²

1.5e+21 pots
```

90% C.L.

3σ

PRELIMINARY
4. Summary

- Three mixing angles have been determined: \( \theta_{12} \approx \pi/6, \theta_{23} \approx \pi/4, \theta_{13} \approx \pi/20. \)

- The remaining parameters to be measured are \( \text{sign}(\Delta m^2_{31}) \), \( \text{sign}(\theta_{23} - \pi/4) \) and \( \delta \).

- To determine \( \delta \), parameter degeneracy (particularly of mass hierarchy) must be resolved.

- Accelerator and reactor experiments are expected to determine \( \text{sign}(\Delta m^2_{31}) \) and \( \delta \) in 10-20 years.
Backup slides
### Global Fits:

<table>
<thead>
<tr>
<th>parameter</th>
<th>best fit ±1σ</th>
<th>best fit ±1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2_{21}$ [$10^{-5}$ eV$^2$]</td>
<td>$7.62 \pm 0.19$</td>
<td>$7.54^{+0.26}_{-0.22}$</td>
</tr>
<tr>
<td>$\Delta m^2_{31}$ [$10^{-3}$ eV$^2$]</td>
<td>$2.53^{+0.08}_{-0.10}$</td>
<td>$2.43^{+0.07}_{-0.09}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.320^{+0.015}_{-0.017}$</td>
<td>$0.307^{+0.018}_{-0.016}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.84^{+0.08}_{-0.05}$</td>
<td>$0.398^{+0.030}_{-0.026}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.026^{+0.003}_{-0.004}$</td>
<td>$0.0245^{+0.0034}_{-0.0031}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.83^{+0.54}_{-0.64} \pi$</td>
<td>$(0.89^{+0.29}_{-0.44}) \pi$</td>
</tr>
<tr>
<td></td>
<td>$0.07 \pi \alpha$</td>
<td>$(0.90^{+0.32}_{-0.43}) \pi$</td>
</tr>
</tbody>
</table>

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Forero, Tortola, Valle
arXiv:1205.4018

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno
arXiv:1205.5254
3 flavor atmospheric $\nu$ oscillations

$$\frac{\Phi(\nu_e)}{\Phi_0(\nu_e)} - 1 \approx P_2 \cdot (r \cdot \cos^2 \theta_{23} - 1)$$

$$-r \cdot \sin \tilde{\theta}_{13} \cdot \cos^2 \tilde{\theta}_{13} \cdot \sin 2\theta_{23} \cdot (\cos \delta \cdot R_2 - \sin \delta \cdot I_2)$$

$$+2 \sin^2 \tilde{\theta}_{13} \cdot (r \cdot \sin^2 \theta_{23} - 1)$$

Normal Hierarchy

Inverted Hierarchy

10 years HK (5.6Mt year) = 248year SK